# UNIT-I <br> LOW - FREQUENCY SINGLE - STAGE AMPLIFIERS 

## Objectives

- To introduce basic structures of MOSFET and BJT amplifier circuits.
- To familiarize with characteristic parameters of amplifiers
- To design and analyse behaviour of BJT \& FET amplifiers at low frequencies.


## Syllabus

MOS Amplifiers, The basic structure, characterizing MOS amplifiers, low - frequency response of common source (CS) amplifier,common source (CS) amplifier with a source resistance, design of CS amplifier, BJT amplifiers, The basic structure, characterizing BJT amplifiers, low - frequency response of the common emitter (CE) amplifier,common emitter (CE) amplifier with an emitter resistance, design of CE amplifier.

## Pre-requisites

- Small-signal models of MOSFET
- Small-signal models of BJT
- Circuit models for amplifiers
- The terms: Frequency response and Bandwidth of an amplifier


## Outcomes:

Students will be able to

- distinguish unilateral and non-unilateral amplifiers
- analyse FET and BJT amplifiers at low frequencies
- design CE and CS amplifiers at low frequencies


## Pre-requisites

- Small - signal equivalent circuit models of MOSFET
$>$ From a signal point of view the FET behaves as a Voltage Controlled current source. It accepts a signal $V_{G S}$ between gate and source and provides a current $g_{m} V_{G S}$, at the drain terminal.
$>$ The input resistance of this controlled source is very high - ideally infinite. The output resistance - i.e., resistance looking in to the drain - also is high.
$>$ Putting all of this together, we get the circuit in as shown in below figurethat represents the small signal model or small signal equivalent circuit.


Fig. Small signal model for the MOSFET neglecting the dependence of $i^{m}$ on $V_{\text {ds }}$ in saturation
$>$ In the analysis of a MOSFET amplifier circuit, transistor can be replaced by the equivalent model. Ideal constant DC voltage sources are replaced by short circuit, and Ideal constant DC current sources are replaced by an open circuit in the small signal model.
$>$ Another approach of the small signal model of amplifier is that assume the drain current in saturation is independent of the drain voltage. From MOSFET characteristics in saturation, we know that the drain current does in fact depend on $\mathrm{V}_{\mathrm{DS}}$ in linear manner. Such linear relation was modeled by a finite resistance $r_{0}$ between drain and source. we represent it as

$$
r_{o}=\frac{\left|V_{A}\right|}{I_{D}}
$$

$>$ The current $\mathrm{I}_{\mathrm{D}}$ is the value of the DC drain current without the channel -length modulation taken into account ; that is,

$$
I_{D}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L} V_{O V}^{2}
$$

- $r_{D}=\frac{1}{2} k_{n} \frac{1}{L}$
$>$ Accuracy of the small signal model can be improved by including $r_{o}$ in parallel with the controlled source.


Fig.Small signal model for MOSFET including the effect of channel -length modulation.
$>$ It is important to note that small signal model parameters $\mathrm{g}_{\mathrm{m}}$ and $r_{o}$ depend on the DC bias point of the MOSFET.
> MOSFET amplifier voltage gain expression

$$
A_{v}=\frac{v_{d}}{v_{g s}}=-g_{m}\left(R_{D} / / r_{o}\right)
$$

> Thus finite output resistance $r_{o}$ results in a reduction in the magnitude of voltage gain.
$>$ All the above analysis performed on an NMOS transistor, apply equally well to PMOS device, except for using $V_{G S}, V_{t}, V_{O V}$, and $V_{A}$ and replacing $k_{n}$ ' with $k_{p}$.

## The T Equivalent Circuit Model

$>$ It is possible to develop an alternate equivalent circuit model for the MOSFET. The development of such model is known as T model.Figure (a) shows the equivalent circuit of MOSFET without $\mathrm{r}_{\mathrm{o}}$.
$>$ In the figure as shown below, we have added a second $g_{m} V_{g s}$ current source in series with the original controlled source. This addition of current source does not change the terminal currents.
$>$ The newly created circuit node, labeled as X , is joined to the gate terminal G. Observe that the gate current doesn't change(remains equal to zero)-thus this connection doesn't alter the terminal characteristics.
$>$ We have a controlled voltage source $g_{m} \nu_{g s}$ connected across its control voltage vgs. We can replace this controlled source by a resistance as long as this resistance draws an equal current as the source. Thus the value of resistance is $v_{g} / g_{m} \nu_{g s}=l / g_{m}$. This replacement is shown in figure (d)
$>$ Observe thatig is still zero, $i_{d}=g_{m} v_{g s}$ and is $=v_{g} g\left(l / g_{m}\right)=\mathrm{g}_{\mathrm{m}} \mathrm{Vgs}_{\mathrm{g}}$, all same as the original model in figure (c)
$>$ Note that the resistance between gate and source, looking into the gate, is infinite. This observation useful in many applications.


Fig.Development of the $T$ equivalent - circuit model for the MOSFET. For simplicity, $r_{o}$ has been omitted but can be added between $D$ and $S$ in the $T$ model of (d).
> In T-model including resistance $\mathrm{r}_{\mathrm{o}}$ between drain and source the circuit is modeled as follows.

(a)

(b)

Fig.(a)The T model of the MOSFET augmented with the drain to source resistance $r_{0}$; (b) An alternative representation of the $\mathbf{T}$ model.

- Small signal equivalent circuit models of BJT

The Hybrid - $\boldsymbol{\pi}$ Model
An equivalent circuit model for the BJT is shown in below fig


Fig. Two slightly different versions of the simplified hybrid $\boldsymbol{- \pi}$ model for the small signal operation of the BJT. The equivalent circuit in (a) represents the BJT as a voltage controlled current source (a transconductance amplifier), and that in (b) represents the BJT as a current controlled current source (a current amplifier).
$>$ Model (a) represents the BJT as a voltage controlled current source and explicitly includes the input resistance looking into the base $r_{\pi}$. The model obviously yields $i_{c}=g_{m} V_{b e}$ and $i_{b}=v_{b e} / r_{\pi}$.
> At the emitter node we have

$$
\begin{aligned}
i_{e} & =\frac{v_{b e}}{r_{\pi}}+g_{m} v_{b e}=\frac{v_{b e}}{r_{\pi}}\left(1+g_{m} r_{\pi}\right) \\
& =\frac{v_{b e}}{r_{\pi}}(1+\beta)=v_{b e} /\left(\frac{r_{\pi}}{1+\beta}\right) \\
& =v_{b e} / r_{e}
\end{aligned}
$$

$>$ A slightly different circuit model(b) can be obtained by expressing the current of the controlled source $\left(\mathrm{g}_{\mathrm{m}} \mathrm{V}_{\mathrm{be}}\right)$ in terms of the base current $\mathrm{i}_{\mathrm{b}}$ (current controlled current source ) as follows:

$$
\begin{aligned}
g_{m} v_{b e} & =g_{m}\left(i_{b} r_{\pi}\right) \\
& =\left(g_{m} r_{\pi}\right) i_{b}=\beta i_{b}
\end{aligned}
$$

$>$ The two models $(\mathrm{a} \& \mathrm{~b})$ are the simplified hybrid $-\pi$ model for the small signal operation for the BJT. The model parameters $\mathrm{g}_{\mathrm{m}}, \mathrm{r}_{\pi}$ depend on the value of the DC bias current Ic.
$>$ Finally, although the models have been developed for an NPN transistor, they apply equally well to a PNP transistor with no change of polarities.

## The T Model

$>$ Although hybrid $-\pi$ model can be used to carry out small signal analysis of all transistor circuits, there are situations in which alternative model shown in below fig, is much more convenient, is called as T model.

(a)

$$
\begin{aligned}
g_{m} & =I_{C} / V_{T} \\
r_{e} & =\frac{V_{T}}{I_{E}}=\frac{\alpha}{g_{m}}
\end{aligned}
$$


(b)

Fig. Two slightly different versions of what is known as the T model of the BJT. The circuit in (a) is a voltage controlled current source representation and that in (b) is a current controlled current source representation. These models explicitly show the emitter resistance $r_{e}$ rather than the base resistance $\mathbf{r}_{\boldsymbol{\pi}}$ featured in the hybrid $-\boldsymbol{\pi}$ model.
$>$ The two versions of T-model shown in above figure. Fig. (a) represents the BJT as a voltage controlled current source with the control voltage being $\mathrm{v}_{\mathrm{be}}$. Here, however, the resistance between base and emitter, looking into the emitter, is explicitly shown.
$>$ From fig(a) we see clearly that the model yields the correct expression for $i_{c}$ and $i_{b}$. For $i_{b}$ we note that at the base node we have

$$
\begin{aligned}
i_{b} & =\frac{v_{b e}}{r_{e}}-g_{m} v_{b e}=\frac{v_{b e}}{r_{e}}\left(1-g_{m} r_{e}\right) \\
& =\frac{v_{b e}}{r_{e}}(1-\alpha)=\frac{v_{b e}}{r_{e}}\left(1-\frac{\beta}{\beta+1}\right) \\
& =\frac{v_{b e}}{(\beta+1) r_{e}}=\frac{v_{b e}}{r_{\pi}}
\end{aligned}
$$

$>$ If in the model of Fig (a) the current of the controlled source is expressed in terms of the emitter current as follows :

$$
\begin{aligned}
g_{m} v_{b e} & =g_{m}\left(i_{e} r_{e}\right) \\
& =\left(g_{m} r_{e}\right) i_{e}=\alpha i_{e}
\end{aligned}
$$

$>$ We obtain alternative T-model shown in Fig (b). Here the BJT is represented as a current controlled current source but with the control signal being $\mathrm{i}_{\mathrm{e}}$.

Irrespective of complexity of the amplifier circuit, circuit can be represented as a building block in a system one must able to characterize, or model its terminal behaviour. In this section we will study simple but effective amplifier models.

## Voltage Amplifiers

$>$ The model consists of Voltage controlled voltage source having a gain of factor of $\mathrm{A}_{\mathrm{vo}}$
$>$ An input resistance $\mathrm{R}_{\mathrm{i}}$ that accounts for amplifier draws an input current from the signal source.
$>$ Output Resistance $\mathrm{R}_{0}$ that accounts for the change in output voltage as the amplifier is called upon to supply output current to load.

(a)

(b)

Fig.a) Circuit model for voltage amplifier, b) Voltage amplifier with input signal source

## load

$>$ A signal voltage source $\mathrm{V}_{\mathrm{s}}$ having a resistance $\mathrm{R}_{\mathrm{s}}$ connected to at the output to a load resistance $\mathrm{R}_{\mathrm{L}}$.
$>$ The non-zero output resistance $\mathrm{R}_{\mathrm{o}}$ causes only a fraction of $\mathrm{A}_{\mathrm{vo}} \mathrm{V}_{\mathrm{i}}$ to appear across the output.
$>$ Using voltage division rule we obtain

$$
v_{o}=A_{w o} v_{i} \frac{R_{L}}{R_{L}+R_{o}}
$$

Thus the voltage gain is given by

$$
A_{v} \equiv \frac{v_{a}}{v_{i}}=A_{v o} \frac{R_{L}}{R_{L}+R_{o}}
$$

$>$ In order not to lose gain in coupling the amplifier output to load, the output resistance $\mathrm{R}_{\mathrm{o}}$ should be much smaller than the load resistance $\mathrm{R}_{\mathrm{L}}$ or to keep the output voltage $\mathrm{v}_{\mathrm{o}}$ as constant as possible.
$>$ An ideal voltage amplifier is one with $\mathrm{R}_{\mathrm{o}}=0, \mathrm{R}_{\mathrm{L}}=\infty$, then $\mathrm{A}_{\mathrm{v}}=\mathrm{A}_{\mathrm{vo}}$. Where, $\mathrm{A}_{\mathrm{vo}}$ is the voltage gain of the unloaded amplifier, or open circuit voltage gain.
$>$ The finite input resistance $\mathrm{R}_{\mathrm{i}}$ introduces another voltage divider action at the input, with the result that only a fraction of the source signal $\mathrm{V}_{\mathrm{s}}$ actually reaches the input terminals of the amplifier; that is

$$
v_{i}=v_{s} \frac{R_{i}}{R_{i}+R_{s}}
$$

$>$ The overall voltage gain $\left(\mathrm{V}_{\mathrm{o}} / \mathrm{V}_{\mathrm{s}}\right)$ can be found

$$
\frac{v_{o}}{v_{s}}=A_{v o} \frac{R_{i}}{R_{i}+R_{s}} \frac{R_{L}}{R_{L}+R_{o}}
$$

## Cascaded Amplifiers

$>$ To meet given amplifier specifications the need often arises to design the amplifier as a cascade of two or more stages. The stages are not usually identical. To illustrate the analysis and design of cascade amplifiers, we consider a practical example.

## Other Amplifier Types

$>$ The below table shows the four amplifier types, their circuit models ,the definition of their gain parameters, and the ideal values of their input and output resistances.


## Frequency Response of Amplifiers

$>$ An important characterization of an amplifier in terms of its response to input sinusoids of different frequencies. Such characterization of amplifier performance is known as the amplifier frequency response.

## Amplifier bandwidth

The band of frequncies over which the gain of the amplifier is almost constant,to within a certain number of decibels(usually 3 dB ), is called the Amplifier bandwidth.


Fig. Typical magnitude response of amplifier.

## SYLLABUS

### 1.1 Low - frequency Single-stage MOS amplifiers

## MOS amplifiers

$>$ In discrete circuits the MOSFET source is usually tied to the substrate, the body effect will be absent. Therefore we shall not take the body effect into account. Also, in some circuits we will neglect $r_{0}$ in order to keep the analysis simple.

### 1.1.1 The Basic Structure

$>$ Figure 1.1 shows the basic circuit to implement the various configurations of discrete-circuit MOS amplifiers. Among the various schemes for biasing discrete MOS amplifiers, the constant-current biasing is employed for its effectiveness and simplicity. Figure 1.1 indicates the dc current and the dc voltages resulting at various nodes.


Fig.1.1: Basic structure of the circuit used to realize single-stage discrete-circuit MOS amplifier configurations.

### 1.1.2 Characterizing Amplifiers

Most of the amplifiers are non-unilateral amplifiers, i.e., they have internal feedback that may cause their input resistance to depend on the load resistance. Similarly, internal feedback may cause the output resistance to depend on the value of the resistance of the signal source feeding the amplifier. A number of remarks are in order:
$>$ The amplifier is shown fed with a signal source having an open-circuit voltage $\mathrm{v}_{\text {sig }}$ and an internal resistance $\mathrm{R}_{\text {sig. }}$. These can be the parameters of an actual signal source or the Thevenin equivalent of the output circuit of another amplifier stage preceding the one under study in a cascade amplifier. Similarly, $\mathrm{R}_{\mathrm{L}}$ can be an actual load resistance or the input resistance of a succeeding amplifier stage in a cascade amplifier.
$>$ Parameters $\mathrm{R}_{\mathrm{t}}, \mathrm{R}_{0}, \mathrm{~A}_{\mathrm{v} 0}, \mathrm{~A}_{\mathrm{is}}$, and $\mathrm{G}_{\mathrm{m}}$ pertain to the amplifier proper; that is, they do not depend on the values of $R_{\text {sig }}$ and $R_{L}$. By contrast, $R_{i n}, R_{\text {out }}, A_{v}, A_{t}, G_{v 0}$, and $G_{v}$ may depend on one or both of $\mathrm{R}_{\text {sig }}$ and $\mathrm{R}_{\mathrm{L}}$. Also, observe the relationships of related pairs of these parameters; for instance,

$$
R_{i}=\left.R_{\text {in }}\right|_{R_{L}=\infty}, \text { and } R_{o}=\left.R_{\text {out }}\right|_{R_{i 4}}=\cdots
$$

$>$ For non-unilateral amplifiers, $\mathrm{R}_{\text {in }}$ may depend on $\mathrm{R}_{\mathrm{L}}$, and Rout may depend on $\mathrm{R}_{\text {sig. }}$. No such dependencies exist for unilateral amplifiers, for which $R_{i n}=R_{i}$ and Rout $=R_{0}$.
$>$ The loading of the amplifier on the signal source is determined by the input resistance $\mathrm{R}_{\text {in }}$. The value of $\mathrm{R}_{\text {in }}$ determines the current that the amplifier draws from the signal source. It also determines the proportion of the signal $\mathrm{v}_{\text {sig }}$ that appears at the input of the amplifier proper (i.e., $\mathrm{v}_{\mathrm{t}}$ ).
$>$ When evaluating the gain $A v$ from the open-circuit value $A_{v 0}, R_{0}$ is the output resistance. This is because Av is based on feeding the amplifier with an ideal voltage signal vt.
$>$ If the overall voltage gain $G_{v}$ is to be calculated from its open-circuit value $G_{v 0}$, the output resistance to use is Rout. This is because $G_{v}$ is based on feeding the amplifier with vsig, which has an internal resistance Rsig.

### 1.1.3 Low - frequency response of the Common-Source (CS) Amplifier

$>$ The common-source ( CS ) or grounded-source configuration is the most widely used of all MOSFET amplifier circuits. A common-source amplifier realized using the circuit of Fig. 1.1 is shown in Fig. 1.2 (a).
$>$ Observe that to establish a signal ground, or an ac ground as it is sometimes called, at the source, we have connected a large capacitor, $C_{\mathrm{s}}$, between the source and ground. This capacitor, usually in the $p F$ range, is required to provide a very small impedance (ideally, zero impedance; i.e., in effect, a short circuit) at all signal frequencies of interest.
$>$ In this way, the signal current passes through $C_{\text {s }}$ to ground and thus bypasses the output resistance of current source $I$ (and any other circuit component that might be connected to the MOSFET source); hence, $C s$ is called a bypass capacitor.
> The lower the signal frequency, the less effective the bypass capacitor becomes. Here we shall assume that $C_{\text {s }}$ is acting as a perfect short circuit and thus is establishing a zero signal voltage at the MOSFET source.
$>$ In order not to disturb the de bias current and voltages, the signal to be amplified, shown as voltage source $v \$ i g$ with an internal resistance $R_{\text {sig }}$, is connected to the gate through a large
capacitor $\mathrm{C}_{\mathrm{C} 1}$. Capacitor $C_{C l}$, known as a coupling capacitor, is required to act as a perfect short circuit at all signal frequencies of interest while blocking dc.
$>$ As the signal frequency is lowered, the impedance of $C_{C l}$ (i.e., $l / j \omega C_{C l}$ ) will increase and its effectiveness as a coupling capacitor will be correspondingly reduced.
$>$ Here we assume $C_{C I}$ is acting as a perfect short circuit as far as the signal is concerned. Before leaving $\mathrm{C}_{\mathrm{C} 1}$, where the signal source can provide an appropriate dc path to ground, the gate can be connected directly to the signal source and both $R_{G}$ and $C_{C l}$ can be dispensed with.
$>$ The voltage signal resulting at the drain is coupled to the load resistance $R_{L}$ via another coupling capacitor $C_{C 2}$. We assume that $C_{C 2}$ acts as a perfect short circuit at all signal frequencies of interest and thus that the output voltage $v_{0}=v_{d}$.
$>$ Note that $R_{L}$ can be either an actual load resistor, to which the amplifier is required to provide its output voltage signal, or it can be the input resistance of another amplifier stage in cases where more than one stage of amplification is needed.
$>$ To determine the terminal characteristics of the CS amplifier-that is, its input resistance, voltage gain, and output resistance-we replace the MOSFET with its small-signal model. The resulting circuit is shown in Fig. 1.2 (b).
$>$ This amplifier is unilateral. Therefore Rin does not depend on $R_{L}$, and thus $R_{i n}=R_{i}$. Also, Rout will not depend on $\mathrm{R}_{\text {sig }}$, and thus $\mathrm{R}_{\text {out }}=\mathrm{R}_{0}$. Analysis of this circuit is straight forward and proceeds in a step-by-step manner, from the signal source to the amplifier load. At the input

$$
\begin{gathered}
i_{g}=0 \\
R_{\text {in }}=R_{G} \\
v_{i}=v_{\text {sig }} \frac{R_{\text {in }}}{R_{\text {in }}+R_{\text {sig }}}=v_{\text {sig }} \frac{R_{G}}{R_{G}+R_{\text {sig }}}
\end{gathered}
$$




Fig. 1.2: (a) Common-source amplifier based on the circuit of Fig. 1.1, (b) Equivalent circuit of the amplifier for small-signal analysis, (c) Small-signal analysis performed directly on the amplifier circuit with the MOSFET model implicitly utilized.
$>$ Usually $R_{G}$ is selected very large (e.g., in the $\mathrm{M} \Omega$ range) with the result that in many applications $R_{C} \gg R_{\text {sig }}$ and

$$
\begin{gathered}
v_{i} \cong v_{\text {sigg }} \\
v_{\mathrm{s} s}=v_{i} \\
v_{o}=-g_{m} v_{\mathrm{gs}}\left(r_{\mathrm{p}}\left\|R_{D}\right\| R_{i}\right)
\end{gathered}
$$

Thus the voltage gain $\mathrm{A}_{v}$ is

$$
A_{v}=-g_{m}\left(r_{g}\left\|R_{D}\right\| R_{L}\right)
$$

and the open-circuit voltage gain $\mathrm{A}_{v o}$ is

$$
A_{o n}=-g_{m}\left(r_{o} \| R_{D}\right)
$$

The overall voltage gain from the signal-source to the load will be

$$
\begin{aligned}
G_{v i} & =\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} A_{v} \\
& =-\frac{R_{G}}{R_{G}+R_{\mathrm{xig}}} g_{m}\left(r_{a}\left\|R_{D}\right\| R_{L}\right)
\end{aligned}
$$

$>$ Finally, to determine the amplifier output resistance $\mathrm{R}_{\text {out }}$ we set $\boldsymbol{v}_{\text {sigto }} 0$; that is, we replace the signal generator $v_{s i g}$ with a short circuit and look back into the output terminal, as indicated in Fig. 1.2.

$$
R_{\text {cut }}=r_{s} \| R_{D}
$$

$>$ Including the output resistance $\mathrm{v}_{0}$ in the analysis of the CS amplifier is straight forward: Since $\mathrm{r}_{0}$ appears between drain and source, it in effect appears in parallel with $R_{D}$. Since it is usually the case that $\mathrm{r}_{0} \gg \mathrm{R}_{\mathrm{D}}$, the effect of $\mathrm{r}_{0}$ will be a slight decrease in the voltage gain and a decrease in Rout.
$>$ Although small-signal equivalent circuit models provide a systematic process for the analysis of any amplifier circuit. Consider the Fig. 1.2 (c) the small-signal analysis of the CS amplifier performed on a somewhat simplified version of the circuit and its equivalent circuit is shown in Fig. 1.2 (b). The CS amplifier has a very high input resistance, a moderately high voltage gain, and a relatively high output resistance.

### 1.1.4 The Common-Source Amplifier with a Source Resistance

$>$ Here a resistance $\mathrm{R}_{\mathrm{S}}$ in the source lead of the common-source amplifier, as shown in Fig. 1.3 (a). The corresponding small-signal equivalent circuit is shown in Fig. 1.3 (b) where we note that the transistor has been replaced by its T equivalent-circuit model.
$>$ The T model is used in preference to the $п$ - model because it makes the analysis simpler. In general, whenever a resistance is connected in the source lead, as for instance in the sourcefollower circuit, the T model is preferred: The source resistance then simply appears in series with the resistance $1 / g_{m}$ which represents the resistance between source and gate, looking into the source.
$>$ We have not included $\mathrm{r}_{0}$ in the equivalent-circuit model. Including $\mathrm{r}_{0}$ would complicate the analysis considerably; $r_{0}$ would connect the output node of the amplifier to the input side and thus would make the amplifier non-unilateral.
$>$ From Fig. 1.3 (b) we see that as in the case of the CS amplifier,

$$
\begin{gathered}
R_{\mathrm{in}}=R_{i}=R_{G} \\
v_{i}=v_{\text {sis }} \frac{R_{G}}{R_{G}+R_{\text {sin }}}
\end{gathered}
$$

> Unlike the CS circuit, however, here $\mathrm{v}_{\mathrm{gs}}$ is only a fraction of $\mathrm{v}_{\mathrm{I}}$. It can be determined from the voltage divider composed of $1 / g_{\mathrm{m}}$ and $\mathrm{R}_{\mathrm{S}}$ that appears across the amplifier input as follows:

$$
v_{s s}=v_{i} \frac{\frac{1}{g_{m}}}{\frac{1}{g_{m}}+R_{S}}=\frac{v_{t}}{1+g_{m} R_{S}}
$$

$>$ Thus we can use the value of $\mathrm{R}_{\mathrm{S}}$ to control the magnitude of the signal vgs and thus ensure that $\mathrm{vgs}_{\mathrm{gs}}$ does not become too large and cause unacceptably high nonlinear distortion. This is the first benefit of including resistor Rs.
$>$ Rs causes the useful bandwidth of the amplifier to be extended. The mechanism by which Rs causes such improvement in amplifier performance is that of negative feedback. The current $i_{d} i_{s}$ equal to the current iflowing in the source lead; thus,

$$
i_{\Delta}=i=\frac{v_{i}}{\frac{1}{g_{n}}+R_{S}}=\frac{g_{m} v_{i}}{1+g_{m} R_{S}}
$$


(b)

Fig. 1.3: (a) Common-source amplifier with a resistance $R_{s}$ in the source lead, (b) Small-signal equivalent circuit with ro neglected.
$>$ Thus including $R_{S}$ reduces $i_{d}$ by the factor $\left(1+g_{m} R_{S}\right)$, since this is the factor relating $v_{g s}$ to $v_{i}$ and the MOSFET produces $i_{d}=g_{m} v g$. The above drain current equation indicates also that the effect of RS can be thought of as reducing the effective $g_{m}$ by the factor $\left(1+g_{m} R s\right)$.
$>$ The output voltage can now be found from

$$
\begin{aligned}
v_{o} & =-i_{d i}\left(R_{b} \| R_{L}\right) \\
& =-\frac{g_{m}\left(R_{D} \| R_{L}\right)}{1+g_{m} R_{s}} v_{j}
\end{aligned}
$$

$>$ Thus the voltage gain is

$$
A_{v i}=-\frac{g_{m}\left(R_{b} \| R_{h}\right)}{1+g_{\Delta n} R_{S}}
$$

And setting $\mathrm{R}_{\mathrm{L}}=\infty$ gives

$$
A_{v o}=-\frac{g_{m} R_{D}}{1+g_{m} R_{S}}
$$

$>$ The overall voltage gain $\mathrm{G}_{0}$ is

$$
G_{v}=-\frac{R_{G}}{R_{G}+R_{\mathrm{sig}}} \frac{g_{m m}\left(R_{D} \| R_{L}\right)}{1+g_{m 2} R_{S}}
$$

$>$ Comparing the above three equations, with their counterparts without $\mathrm{R}_{\mathrm{S}}$ indicates that including $\mathrm{R}_{\mathrm{S}}$ results in a gain reduction by the factor $\left(1+\mathrm{g}_{\mathrm{m}} \mathrm{R}_{\mathrm{S}}\right)$. This factor is called the amount of feedback and that it determines both the magnitude of performance improvements and, the reduction in gain.
$>$ A resistance $\mathrm{R}_{\mathrm{S}}$ in the source lead increases dc bias stability; that is, $\mathrm{R}_{\mathrm{S}}$ reduces the variability in $I_{D}$. The action of $R_{S}$ that reduces the variability of $I_{D} . R s$ in the circuit of Fig. 1.3 is reducing $i_{d}$. Because of its action in reducing the gain, $\mathrm{R}_{\mathrm{S}}$ is called source degeneration resistance.
$>$ Another useful interpretation of the voltage gain expression is that the gain from gate to drain is simply the ratio of the total resistance in the drain, ( $\mathrm{R}_{\mathrm{D}}$ in parallel with $\mathrm{R}_{\mathrm{L}}$ ), to the total resistance in the source, $\left[\left(1 / \mathrm{gm}_{\mathrm{m}}\right)+\mathrm{Rs}\right]$.

### 1.2 BJT Amplifiers

$>$ There are three basic configurations for single-stage BJT amplifiers:

- Common-Emitter
- Common-Base
- Common-Collector


### 1.2.1 The basic structure of BJT amplifier

> The basic circuit that utilize to implement the various configurations of BJT amplifiers with constant current biasing is shown in figure 1.4. The value of $\mathrm{R}_{\mathrm{B}}$ is selected as large value in order to keep the input resistance at the base large. However it is also required to limit the dc voltage drop across $\mathrm{R}_{\mathrm{B}}$


Fig. 1.4: Basic structure of the circuit used to realize single-stage, discrete-circuit BJT amplifier configurations

### 1.2.2 Characterizing BJT amplifiers

$>$ To use BJT amplifier circuits, it is important to know how to characterize the performance of amplifiers as circuit building blocks. These amplifiers have internal feedback that may cause their input resistance to depend on the load resistance. Similarly internal feedback may cause the output resistance to depend on the value of the resistance of the signal source feeding the amplifier.

## Circuit



## Definitions

篦 Input resistance with no load：
$\left.R_{i} \equiv \frac{v_{i}}{i_{i}}\right|_{R_{L}=\infty}$
Input resistance：
$R_{\mathrm{in}} \equiv \frac{v_{i}}{i_{i}}$
（ Open－circuit voltage gain：
$\left.A_{v o} \equiv \frac{v_{o}}{v_{i}}\right|_{R_{L}=\infty}$
氥 Voltage gain：
$A_{v} \equiv \frac{v_{o}}{v_{i}}$
触 Short－circuit current gain：
$\left.A_{i s} \equiv \frac{i_{o}}{i_{i}}\right|_{R_{L}=0}$
襁 Current gain：
$A_{i} \equiv \frac{i_{o}}{i_{i}}$
－Short－circuit transconductance：
$\left.G_{m} \equiv \frac{i_{o}}{v_{i}}\right|_{R_{L}=0}$
－Output resistance of amplifier proper：
$\left.R_{o} \equiv \frac{v_{x}}{i_{x}}\right|_{v_{i}=0}$

Output resistance：

$$
\left.R_{\text {out }} \equiv \frac{v_{x}}{i_{x}}\right|_{v_{\text {sig }}=0}
$$


－Open－circuit overall voltage gain：
$\left.G_{v o} \equiv \frac{v_{o}}{v_{\mathrm{sig}}}\right|_{R_{L=\infty}=\infty}$
䌵 Overall voltage gain：
$G_{v} \equiv \frac{v_{o}}{v_{\mathrm{sig}}}$
( A :


路 B :

(


## Relationships

$$
\begin{array}{lll}
\frac{v_{i}}{v_{\mathrm{sig}}}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} & G_{v}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} A_{v o} \frac{R_{L}}{R_{L}+R_{o}} \\
A_{v}=A_{v o} \frac{R_{L}}{R_{L}+R_{o}} & G_{v o}=\frac{R_{i}}{R_{i}+R_{\mathrm{sig}}} A_{v o} \\
A_{v o}=G_{m} R_{o} & G_{v}=G_{v o} \frac{R_{L}}{R_{L}+R_{\text {out }}}
\end{array}
$$

1. The amplifier should fed with a signal source having an open circuit voltage $\mathrm{v}_{\text {sig }}$ and an internal resistance $\mathrm{R}_{\text {sig. }}$. These can be the parameters of an actual signal source or the Thevenin equivalent of the output circuit of another amplifier stage in a cascade amplifier stage. Similarly, $\mathrm{R}_{\mathrm{L}}$ be the actual load resistance or the input resistance of a succeeding stage in a cascading amplifier.
2. Parameters $\mathrm{R}_{i}, \mathrm{R}_{o}, \mathrm{~A}_{v o}, \mathrm{~A}_{i s}$ and $\mathrm{G}_{m}$ pertain to the amplifier proper; i.e., they do not depend on the values of $\mathrm{R}_{s i g}$ and $\mathrm{R}_{L} . \mathrm{R}_{i n}, \mathrm{R}_{o u}, \mathrm{~A}_{v}, \mathrm{~A}_{i}, \mathrm{G}_{v o}$ and $\mathrm{G}_{v}$ may depend on the values of $\mathrm{R}_{s i g}$ and $\mathrm{R}_{L}$. Relationships of related pairs: $\mathrm{R}_{i}=\mathrm{R}_{i n} \mid \mathrm{R}_{L}=\infty$ and $\mathrm{R}_{o}=\mathrm{R}_{\text {oul }} \mid \mathrm{R}_{\text {sig }}=0$
3. For non-unilateral amplifiers, $\mathrm{R}_{\text {in }}$ may depends on $\mathrm{R}_{L}$ and $\mathrm{R}_{\text {ou }}$ depends on $\mathrm{R}_{\text {sig }}$. No such dependencies exists for unilateral amplifiers, for which $\mathrm{R}_{\text {in }}=\mathrm{R}_{i}$ and $\mathrm{R}_{\text {out }}=\mathrm{R}_{o}$
4. The loading of the amplifier on the signal source is determined by the input resistance $\mathrm{R}_{i n}$. The value of $\mathrm{R}_{\text {in }}$ determines the current that the amplifier draws from the signal source. It also determines the proportion of the signal $v_{s i g}$ that appears at theinput of the amplifier proper, that is, $v_{i}$.
5. When evaluating the gain $A v$ from the open-circuit value $A_{v o}, R_{0}$ is the output resistance to use. This is because A ,, is based on feeding the amplifier with an ideal voltagesignal $v_{i}$.

On the otherhand, if we are evaluating the overall voltage gain $G_{v}$ from its open-circuit value $G_{v}$ the output resistance to use is $\mathrm{R}_{\text {out }}$. This is because $G_{v}$ is based on feeding the amplifierwith $v_{\text {sig }}$, which has an internal resistance $v_{\text {sig }}$.

### 1.2.3 Low - frequency response of the Common-Emitter Amplifier

$>$ First, assume $\mathrm{Re}=0$ (this is not re, but an explicit resistor)
$>$ The BJT is biased with a current source (with high output impedance) and a capacitor connects the emitter to ground.Capacitor provides an AC short at the emitter for small time-varying signals but is an open circuit for DC signals
$>$ Can redraw the circuit with an equivalent circuit that replaces the BJT with its hybrid- $\pi$ model


Fig. 1.5: a) CE Amplifier, b) Equivalent circuit by replacing BJT with $\pi$-model
$>$ The terminal characteristics of amplifier are analysed as

$$
R_{\mathrm{in}} \equiv \frac{v_{i}}{i_{i}}=R_{B} \| R_{i b}
$$

where $R_{i b}$ is the input resistance looking into the base. Since the emitter is grounded,

$$
R_{i b}=r_{\pi}
$$

Normally we select $\mathrm{R}_{\mathrm{B}} \gg r_{\pi}$, which results $\mathrm{R}_{\mathrm{in}} \equiv r_{\pi}$
$>$ The fraction of source signal will appear across the input terminals of the amplifier and is given by,

$$
\begin{aligned}
v_{l} & =v_{\text {sig }} \frac{R_{\text {in }}}{R_{\text {in }}+R_{\text {sig }}} \\
& =v_{\text {sig }} \frac{\left(R_{B} \| r_{\pi}\right)}{\left(R_{B}+r_{\pi}\right)+R_{\text {sig }}}
\end{aligned}
$$

which for $R_{B}>r_{\pi}$ becomes

Next we note that

$$
v_{i} \cong v_{\text {sig }} \frac{r_{\pi}}{r_{\pi}+R_{\text {sig }}}
$$

$$
v_{\pi}=v_{i}
$$

At the output of the amplifier we have

$$
v_{o}=-g_{m} v_{\pi}\left(r_{o}\left\|R_{C}\right\| R_{L}\right)
$$

> The voltage gain of the amplifier is

$$
A_{v}=-g_{m}\left(r_{o}\left\|R_{C}\right\| R_{L}\right)
$$

$>$ The open circuit voltage gain is obtained by making $\mathrm{R}_{\mathrm{L}}=\infty$

$$
A_{v o}=-g_{m}\left(r_{o} \| R_{C}\right)
$$

from which we can say that the effect of $\mathrm{r}_{0}$ is simply to reduce the gain, usually $\mathrm{r}_{0} \gg \mathrm{R}_{\mathrm{C}}$, results in

$$
A_{v o} \cong-g_{m} R_{C}
$$

The output resistnace $\mathrm{R}_{\text {out }}$ can be found from the equivalent circuit 1.5 (b) by looking back into the output terminal while short-circuiting the source $v_{\text {sig. }}$. This will result in $v_{\pi}=0$, and

$$
R_{\text {out }}=R_{C} \| r_{o}
$$

Thus $r_{0}$ reduces the output resistance of the amplifier, typically $r_{0} \gg R_{C}$ and

$$
R_{\mathrm{out}} \cong R_{C}
$$

For this unilateral amplifier $\mathrm{R}_{0}=\mathrm{R}_{\mathrm{out}}$, we can use $\mathrm{A}_{\mathrm{v0} 0}$ and $\mathrm{R}_{0}$ to obtain the voltage gain $\mathrm{A}_{\nu}$ corresponding to any particular $\mathrm{R}_{\mathrm{L}}$,

$$
A_{v}=A_{50} \frac{R_{L}}{R_{L}+R_{o}}
$$

The overall voltage gain from source to load, $\mathrm{G}_{\nu}$ can be obtained by multiplying $\left(\mathrm{v}_{\mathrm{i}} / \mathrm{v}_{\text {sig }}\right)$ by $\mathrm{A}_{v}$,

$$
G_{v}=\frac{\left(R_{B} \| r_{\pi}\right)}{\left(R_{B} \| r_{\pi}\right)+R_{\text {isg }}} g_{m}\left(r_{o}\left\|R_{C}\right\| R_{L}\right)
$$

For the case $\mathrm{R}_{\mathrm{B}} \gg r_{\pi}$, this expression simplifies to

$$
G_{v} \cong-\frac{\beta\left(R_{C}\left\|R_{L}\right\| r_{o}\right)}{r_{\pi}+R_{\text {dig }}}
$$

And voltage gain reduces to

$$
G_{v} \cong-g_{m}\left(R_{C}\left\|R_{L}\right\| r_{o}\right)
$$

$>$ The short circuit current gain is calculated as

$$
i_{o t}=-g_{m} v_{n}
$$

Since $v_{\pi}$ is related to $i_{i}$ by

$$
v_{n}=v_{i}=i_{i} R_{\text {in }}
$$

The short-circuit current gain can be found as

$$
A_{i b}=\frac{i_{\text {eq }}}{i_{i}}=-g_{m} R_{\text {in }}
$$

Substituting $\mathrm{R}_{\mathrm{in}}=\mathrm{R}_{\mathrm{B}} \| r_{\pi}$, we can see that in the case $\mathrm{R}_{\mathrm{B}} \gg r_{\pi},\left|\mathrm{A}_{i s}\right|$ reduces to $\beta$, which is to be expected since $\beta$ is the short - circuit current gain of the common - emitter configuration.

Conclusion:the common emitter configuration can provide large voltage gain, large current gain, low input resistance and relatively high output resistance.

### 1.2.4 Common Emitter Amplifier with Emitter Resistance

Characteristics with $\mathrm{Re}_{\mathrm{e}}$

- gain is less with but less dependent on $\beta$
- input resistance is higher
- allows higher input signal voltage


Fig. 1.6: a) CE Amplifier with emitter resistor, b) Equivalent circuit by replacing BJT with Tmodel

T determine the amplifier input resistance $\mathrm{R}_{\text {in }}$, form the equivalent circuit shown in figure 1.6 (b), at the input $\mathrm{R}_{\mathrm{in}}$ is the parallel equivalent of $\mathrm{R}_{\mathrm{B}}$ and the input resistance at the base $\mathrm{R}_{\mathrm{ib}}$.

$$
R_{\mathrm{in}}=R_{B} \| R_{i b}
$$

The input resistance at the base $R_{i b}$ can be found from

$$
R_{i b} \equiv \frac{v_{i}}{i_{b}}
$$

where

$$
i_{b}=(1-\alpha) i_{e}=\frac{i_{e}}{\beta+1}
$$

and

$$
i_{e}=\frac{v_{i}}{r_{e}+R_{e}}
$$

Thus,

$$
R_{i b}=(\beta+1)\left(r_{e}+R_{e}\right)
$$

It says that the input resistance looking into the base is $(1+\beta)$ times the total resistance in the emitter. Multiplication by the factor $(1+\beta)$ is known as resistance - reflection rule.

$$
\begin{aligned}
\frac{R_{i b}\left(\text { with } R_{e} \text { included }\right)}{R_{i b}\left(\text { without } R_{e}\right)} & =\frac{(\beta+1)\left(r_{e}+R_{e}\right)}{(\beta+1) r_{e}} \\
& =1+\frac{R_{e}}{r_{e}} \cong 1+g_{m} R_{e}
\end{aligned}
$$

To determine the voltage gain $A_{v}$,

$$
\begin{aligned}
v_{o} & =-i_{c}\left(R_{C} \| R_{L}\right) \\
& =-\alpha i_{e}\left(R_{C} \| R_{L}\right) \\
& A_{v} \equiv \frac{v_{o}}{v_{i}}=-\frac{\alpha\left(R_{C} \| R_{L}\right)}{r_{e}+R_{e}}
\end{aligned}
$$

Since $\alpha \cong 1$,

$$
A_{v} \cong-\frac{R_{C} \| R_{L}}{r_{e}+R_{e}}
$$

The open circuit voltage gain is given by

$$
A_{v o}=-\frac{\alpha R_{C}}{r_{e}+R_{e}}
$$

which can be expressed alternatively as

$$
\begin{gathered}
A_{v o}=-\frac{\alpha}{r_{e}} \frac{R_{C}}{1+R_{e} / r_{e}} \\
A_{v o}=-\frac{g_{m} R_{C}}{1+\left(R_{e} / r_{e}\right)} \cong-\frac{g_{m} R_{C}}{1+g_{m} R_{e}}
\end{gathered}
$$

Including $R_{e}$ thus reduces the voltage gain by the factor ( $1+g_{m} R_{e}$ ), which is the same factor by which $\mathrm{R}_{\mathrm{ib}}$ is increased.

The output resistance $\mathrm{R}_{\text {out }}$ can be found from the circuit, $\mathrm{R}_{\text {out }}=\mathrm{R}_{\mathrm{C}}$.

The short - circuit current gain $\mathrm{A}_{\text {is }}$ can be found from the circuit

$$
\begin{aligned}
i_{o s} & =-\alpha i_{e} \\
i_{i} & =v_{i} / R_{\mathrm{in}} \\
A_{i s} & =-\frac{\alpha R_{\mathrm{in}} i_{e}}{v_{i}}
\end{aligned}
$$

$$
A_{i s}=-\frac{\alpha\left(R_{B} \| R_{i b}\right)}{r_{e}+R_{e}}
$$

which for the case $R_{B} \gtrdot R_{i b}$ reduces to

$$
A_{i s}=\frac{-\alpha(\beta+1)\left(r_{e}+R_{e}\right)}{r_{e}+R_{e}}=-\beta
$$

the same value as for the CE circuit.

The overall voltage gain from source to load can be obtained by multiplying $A_{v}$ by $\left(\mathrm{v}_{\mathrm{i}} / \mathrm{v}_{\text {sig }}\right)$

$$
G_{v}=\frac{v_{i}}{v_{\text {sig }}} \cdot A_{v}=-\frac{R_{\text {in }}}{R_{\text {sig }}+R_{\text {in }}} \frac{\alpha\left(R_{C} \| R_{L}\right)}{r_{e}+R_{e}}
$$

Substituting for $R_{\mathrm{in}}$ by $R_{B} \| R_{i b}$, assuming that $R_{B} \geqslant R_{i b}$, and substituting for $R_{i b}$

$$
G_{v} \cong-\frac{\beta\left(R_{C} \| R_{L}\right)}{R_{\text {sig }}+(\beta+1)\left(r_{e}+R_{e}\right)}
$$

This gain is lower than that of the CE amplifier because of the additional term $(1+\beta) \mathrm{R}_{\mathrm{e}}$ in the denominator.

$$
\frac{v_{\pi}}{v_{i}}=\frac{r_{e}}{r_{e}+R_{e}} \cong \frac{1}{1+g_{m} R_{e}}
$$

Thus, for the same $v_{\pi}$, the signal at the input terminal of the amplifier, $v_{i}$, can be greater than for the $C E$ amplifier by the factor $\left(1+g_{m} R_{e}\right)$.

## Conclusion:

- The input resistance $\mathrm{R}_{\mathrm{ib}}$ is increased by the factor $\left(1+\mathrm{g}_{\mathrm{m}} \mathrm{R}_{\mathrm{e}}\right)$.
- The voltage gain from base to collector, $A_{v}$ is reduced by the factor $\left(1+\mathrm{g}_{\mathrm{m}} \mathrm{R}_{\mathrm{e}}\right)$.
- For the same nonlinear distortion, the input signal $v_{i}$ can be increased by the factor $\left(1+g_{\mathrm{m}} \mathrm{R}_{\mathrm{e}}\right)$.
- The overall voltage gain is less dependent on the value of $\beta$.
- The high - frequency response is significantly improved.


## UNIT-I

Assignment-Cum-Tutorial Questions

## A. Objective Questions

1. Which of the following is true in case of unilateral amplifiers
a) Input resistance doesn't depend on $\mathrm{R}_{\mathrm{L}}$
b) Output resistance doesn't depend on $\mathrm{R}_{\text {sig }}$
c) Both a \& b
d) None of the above.
2. All practical amplifiers are
a) Unilateral
b) Non-unilateral
c) May be sometimes unilateral
d)None
3. The phase difference between the output and input voltages of a CE amplifier is
a) $180^{\circ}$
b) $0^{\circ}$
c) $90^{\circ}$
d) $270^{\circ}$
4. The purpose of capacitors in a transistor amplifier is to ............ [ ]
a) Protect the transistor
b) Cool the transistor
b) Couple or bypass a.c. component
d)Provide biasing
5. If the input capacitor of a transistor amplifier is short-circuited, then.
a) Transistor will be destroyed
b) Biasing conditions will change
c)Signal will not reach the base
d) None of the above
6. The amplifier works as a linear system for $\qquad$
a) High frequency signals
b) Low frequency signals
c)Small signals
d) large signals
7. State the differences between Unilateral and Non Unilateral Amplifiers
8. Draw the CS Amplifier and its equivalent circuit for small- signal analysis.
9. Draw the CS Amplifier with source resistance and its equivalent circuit for small- signal analysis
10. Why Source Resistance in CS amplifier also called as "Source degeneration" resistance.
11. Define Bandwidth and Figure of Merit of an amplifier?
12. Draw the basic structure of the circuit used to realize single-stage discretecircuit BJT amplifier configurations.
13. Draw the frequency response curve of an amplifier?
14. Draw the basic structure of the circuit used to realize single-stage discretecircuit MOS amplifier configurations.

## B.Subjective Questions

1. What are the characteristic parameters of amplifiers? Define $\&$ provide relationships.
2. Explain the Small signal operation of CS amplifier. Find the expressions for input resistance ( $\mathrm{R}_{\mathrm{in}}$ ), Output resistance ( $\mathrm{R}_{\mathrm{out}}$ ), voltage gain $\left(\mathrm{A}_{\mathrm{v}}\right)$ and overall voltage gain $\left(\mathrm{G}_{\mathrm{v}}\right)$.
3. Analyse the CS amplifier with source degeneration resistance to find the expressions for $R_{i n}, R_{o u t}, A_{v}$ and $G_{v}$.
4. Analyse the CE amplifier with emitter degeneration resistance to find the expressions for $R_{i n}$, Rout, $A_{v}, i_{o s}$ and $G_{v}$.
5. Explain the small signal operation of CE amplifier \& deduce the expressions for $R_{\text {in }}, R_{\text {out }}, A_{v}, i_{\text {os }}$ and $G_{v}$.
6. Considering a CS amplifier, find $\mathrm{R}_{\mathrm{in}}, \mathrm{A}_{\mathrm{vo}}$, and $\mathrm{R}_{\mathrm{out}}$, both without and with $\mathrm{r}_{\mathrm{o}}$ taken into account. Then calculate the overall voltage gain $G_{v}$, with $r_{o}$ taken into account, for the case $R_{\text {sig }}=100 \mathrm{k} \Omega, \mathrm{R}_{\mathrm{L}}=15 \mathrm{k} \Omega$. If $\mathrm{v}_{\text {sig }}$ is a 0.4 V Peak to peak sinusoid, what output signal $v_{o}$ results?
7. Calculate the overall voltage gain $G_{v}$ of a common source amplifier for which $g_{m}$ $=2 \mathrm{~mA} / \mathrm{V}, \mathrm{r}_{0}=50 \mathrm{k} \Omega, \mathrm{R}_{\mathrm{D}}=10 \mathrm{~K} \Omega$, and $\mathrm{R}_{\mathrm{G}}=10 \mathrm{M} \Omega$. The amplifier is fed from a signal source with a Thevenin resistance of $0.5 \mathrm{M} \Omega$, and the amplifier output is coupled to a load resistance of $20 \mathrm{~K} \Omega$.
8. A CS amplifier using an NMOS transistor biased in the manner as shown in below figure for which $g_{m}=2 \mathrm{~mA} / \mathrm{V}$ is found to have an overall voltage gain $\mathrm{G}_{0}$ of $-16 \mathrm{~V} / \mathrm{V}$. What value should a resistance $\mathrm{R}_{\mathrm{S}}$ inserted in the source lead have to reduce the voltage gain by a factor of 4 ?

9. The overall voltage gain of the amplifier as shown in below figure was measured with a resistance $R_{s}$ of $1 \mathrm{~K} \Omega$ in place and found to be $-10 \mathrm{~V} / \mathrm{V}$. When $\mathrm{R}_{\mathrm{s}}$ is shorted, but the circuit operation remained linear the gain doubled. What must $g_{m}$ be? What value of $R_{s}$ is needed to obtain an overall voltage gain of- $8 \mathrm{~V} / \mathrm{V}$ ?

10. A BJT amplifier is measured to have $\mathrm{R}_{\mathrm{i}}=10 \mathrm{k} \Omega . A_{\nu 0}=100 \mathrm{~V} / \mathrm{V}$, and $R_{0}=100 \boldsymbol{\Omega}$. Also, when a load resistance $R_{L}$ of $1 \mathrm{k} \Omega$ is connected between the output terminals, the input resistance is found to decrease to $8 \mathrm{k} \Omega$. If the amplifier is fedwith a signal source having an internal resistance of $2 \mathrm{k} \Omega$, find $G_{m}, A_{v}, G_{v}, G_{v}, R_{o u t}$, and $A_{i}$.
11. In the circuit shown below, $\mathrm{v}_{\text {sig }}$ is a small sinewavesignal with zero average. The transistor $\beta$ is 100 .
(a) Find the value of $R_{E}$ to establish a dc emitter current ofabout 0.5 mA .
(b) Find $\mathrm{R}_{\mathrm{C}}$ to establish a dc collector voltage of about +5 V .
(c) For $R_{\mathrm{L}}=10 \mathrm{k} \Omega$ and the transistor $\mathrm{r}_{0}=200 \mathrm{k} \Omega$, draw thesmall-signal equivalent circuit of the amplifier and determineits overall voltage gain.


## C. GATE questions

1. A small signal source $V_{i}(t)=(A \cos 20 t+B \sin 10 t)$ is applied to a transistor amplifieras shown below. The transistorhas $\beta=150$ and $h_{i e}=3 \Omega$. Which expression bestapproximate $\mathrm{V}_{0}(\mathrm{t})$ ?
(GATE 2009)

(A) $\mathrm{V}_{0}(\mathrm{t})=1500(\mathrm{~A} \cos 20 \mathrm{t}+\mathrm{B} \sin 10 \mathrm{t})$
(B) $\mathrm{V}_{0}(\mathrm{t})=1500(\mathrm{Acos} 20 \mathrm{t}+\mathrm{B} \sin 10 \mathrm{t})$
(C) $\mathrm{V}_{0}(\mathrm{t})=1500 \mathrm{Bsin} 10 \mathrm{t}$
(D) $\mathrm{V}_{\mathrm{o}}(\mathrm{t})=150 B \sin 10 \mathrm{t}$
2. The amplifier circuit shown below uses a silicon transistor. The capacitors $C_{c}$ and $C_{E}$ can be assumed to be short at signal frequency and effect of output Resistancero can be ignored. If $C_{E}$ is disconnected from the circuit, which one of the followingstatements is true? (GATE 2010)

(A) The input resistance $R_{i}$ increases and magnitude of voltage gain $A_{v}$ decreases
(B) The input resistance $R_{i}$ decreases and magnitude of voltage gain $A_{V}$ increases
(C) Both input resistance Riand magnitude of voltage gain Avdecreases
(D) Both input resistance $R_{\text {a and }}$ the magnitude of voltage gain $A_{\text {vincreases }}$
3. Consider the common emitter amplifier shown below with the following circuit
parameters:
$\beta=100, g_{m}=0.3861 \mathrm{~A} / \mathrm{V}, r_{0}=259 \Omega, R_{s}=1 \mathrm{k} \Omega, R_{B}=93 \mathrm{k} \Omega, R c=250 \mathrm{k} \Omega, R_{l}=1 \mathrm{k} \Omega, C_{1}=3 \mu \mathrm{Fand} C_{2}=4.7 \mu \mathrm{~F}$


The resistance seen by the source $\mathrm{v}_{\mathrm{s}}$ is $\qquad$
(A) $258 \Omega$
(B) $1258 \Omega$
(C) $93 \mathrm{k} \Omega$
(D) $\infty$

# Unit - II <br> High - frequency single - stage amplifiers 

## Objectives

- To Discuss the General considerations of High - frequency Response of an amplifier.
- To Discuss the High-frequency response of the CS and CE amplifiers
- Analysis CS and CE of amplifiers using Miller's theorem
- To analyze, and characterize the different Cascode Amplifiers like MOS Cascode and BJT Cascode
- To analyze, and characterize the Darlington configuration.


## Syllabus

General considerations, high-frequency response of CS and CE amplifiers, cascade amplifier, Darlington configuration.

## Pre-requisites

- The MOSFET internal Capacitances
- The High-Frequency MOSFET Model
- The BJT Internal Capacitances and High Frequency Model
- Circuit models for amplifiers
- The terms: Frequency response and Bandwidth of an amplifier


## Outcomes:

Students will be able to
Draw, analyse, and characterize the MOS and BJT single-stage amplifiers at high frequencies.
Draw, analyze, and characterize - Cascode, Darlington,

## Pre-requisites

THE MOSFET INTERNAL CAPACITANCES :
There are basically two types of internal capacitances in the MOSFET

1. The gate capacitive effect: The gate electrode (polysilicon) forms a parallel-plate capacitor with the channel, with the oxide layer serving as the capacitor dielectric. W e discussed the gate (or oxide) capacitance in previous sections and denoted its value per unit area as Cox.
2. The source-body and drain-body depletion-layer capacitances: These are the capacitances of the reverse-biased $p n$ junctions formed by the $n+$ source region (also called the diffusion) and the p-type substrate and by the $n+$ drain region (the drain diffusion) and the substrate.

## The Junction Capacitances :

The depletion-layer capacitances of the two reverse-biased junctions formed between each of the source and the drain diffusions and the body can $b$ e determined using the formula developed in previous Section. Thus, for the source diffusion, we have the source body capacitance, Csb,

$$
C_{s b}=\frac{C_{s b 0}}{\sqrt{1+\frac{V_{S B}}{V_{0}}}}
$$

here $C s b_{0}$ is the value of $C s b$ at zero body-source bias, $V S B$ is the magnitude of the reverse bias voltage, and $V_{0}$ is the junction built-in voltage ( 0.6 V to 0.8 V ). Similarly, for the drain diffusion, we have the drain-body capacitance $C d b$,

$$
C_{d b}=\frac{C_{d s 0}}{\sqrt{1+\frac{V_{D B}}{V_{0}}}}
$$

## The High-Frequency MOSFET Model:

The below figure shows the high frequency model of MOSFET by adding all the capacitances.


Figure 1a: High-frequency equivalent circuit model for the MOSFET.


Figure 1b: The equivalent circuit for the case in which the source is connected to the substrate (body),


Figure 1c: The equivalent circuit model of (b) with $C d b$ neglected (to simplify analysis).

## The MOSFET Unity-Gain Frequency ( $f_{\mathrm{T}}$ ):



Figure 2: Circuit for determining the short-circuit current gain $I_{o} / l_{\text {i }}$.
A figure of merit for the high-frequency operation of the MOSFET as an amplifier is the unity-gain frequency, $\mathrm{f}_{\mathrm{T}}$. This is defined as the frequency at which the short-circuit current gain of the common-source configuration becomes unity. The above figure shows the MOSFET hybrid $\Pi^{-}$model with the source as the common terminal between the input and output ports. To determine the short-circuit current gain, the input is fed with a current-source signal $\mathrm{I}_{\mathrm{i}} \cdot$ and the output terminals are short-circuited. It is easy to see that the current in the short circuit is given by

$$
I_{o}=g_{m} V_{g s}-s C_{8 d} V_{g s}
$$

let us assume $\mathrm{C}_{\mathrm{gd}}$ is a very small value and it can be neglected w e can express $V g s$ in terms of the input current $I_{i}$ as

$$
V_{g s}=I_{i} / s\left(C_{g s}+C_{g d}\right)
$$

By the above two equations we can get the short circuit current gain as

$$
\frac{I_{o}}{I_{i}}=\frac{g_{m}}{s\left(C_{g s}+C_{g \delta}\right)}
$$

For physical frequencies $S=J W$, it can be seen that the magnitude of the current gain becomes unity at the frequency

$$
0_{I} \equiv g_{m} /\left(C_{g h}+C_{k d}\right)
$$

Thus the unity-gain frequency $f_{\mathrm{T}}$ is given as

$$
f_{T}=\frac{g_{m}}{2 \pi\left(C_{z_{s}}+C_{n d}\right)}
$$

Hence $f_{\mathrm{T}}$ is proportional to $g m$ and inversely proportional to the FET internal capacitances.

## The BJT Internal Capacitances and High Frequency Model

The high frequency response of BJT amplifiers rolls off due to short circuiting effect of internal capacitances. These internal capacitances are resulting from charge- storage effects.

The High frequency Hybrid- $\pi$ model is shown as below.


Fig. The High- Frequency Hybrid - $\pi$ model
$\mathrm{r}_{\mathrm{x}}$ : Base Spreading Resistance :
It is the resistance between the external base terminal B and internal node B' (also called as implicit node, bulk node ). Its typical value is $100 \Omega$. negligible at low frequencies but effective at high frequencies.
$r_{\Pi}$ : It is the resistnace looking into the base terminal.
typical value is $1 \mathrm{~K} \Omega$.

There are two sources of internal capacitances effect

1. Base- Emitter Capacitance $\mathrm{C} \pi$
2. Collector-Base Junction Capacitance $\mathrm{C} \mu$

## 1. Base- Emitter Capacitance $\mathbf{C} \boldsymbol{\pi}$ :

$C \pi$ is the sum of two capacitances.Cde Emitter- Base Diffusion Capacitance
Cje Emitter - Base Junction Capacitance.

$$
\mathrm{C} \pi=\mathrm{Cde}+\mathrm{Cje}
$$

i) Cde Emitter- Base Diffusion Capacitance: It is called as Base - Charging or Diffusion Capacitance Cde.

It is due to minority carrier storage in the base region when the transistor is operating in the active or satuaration mode. i.e when Emitter- Base Junction (EBJ) is forward biased.
$\mathrm{Cde}=\mathrm{dQ} / \mathrm{dv}_{\mathrm{BE}}$
considering, an npn transistor in active region..

$$
\begin{equation*}
i_{C}=I_{S} e^{v_{B E} / V_{T}} \tag{1}
\end{equation*}
$$

$$
\begin{align*}
I_{S} & =\frac{A_{E} q D_{n} n_{i}^{2}}{N_{A} W}  \tag{2}\\
Q_{n} & =\frac{A_{E} q W n_{i}^{2}}{2 N_{A}} e^{v_{B E} / V_{T}} \tag{3}
\end{align*}
$$

Using 1, 2, 3

$$
Q_{n}=\frac{W^{2}}{2 D_{n}} i_{C}=\tau_{F} i_{C}
$$

where,

$$
\tau_{F}=\frac{W^{2}}{2 D_{n}}
$$

$\tau_{\mathrm{F}}$ is known as the forward base- transit time. It represents the average time a charge carrier (electron) spends in crossing the base.

$$
\begin{gathered}
C_{d e} \equiv \frac{d Q_{n}}{d v_{B E}} \\
=\tau_{F} \frac{d i_{C}}{d v_{B E}} \\
C_{d e}=\tau_{F} g_{m}=\tau_{F} \frac{I_{C}}{V_{T}}
\end{gathered}
$$

## ii :Cje :: Base - Emitter Junction Capacitance

$$
C_{j e}=\frac{C_{j e 0}}{\left(1-\frac{V_{B E}}{V_{0 e}}\right)^{m}}
$$

Where, Cje 0 is the value of $\mathrm{Cjeat} \mathrm{VBE}=0 \mathrm{~V}$.
$\mathrm{V}_{0 \mathrm{e}}$ is the EBJ built in voltage $(0.7-0.9 \mathrm{~V})$
$\mathrm{m}=$ grading coefficient , typically $1 / 2$.
For a forward biased EBJ in the active mode, $\mathrm{Cje}=2 \mathrm{Cje} 0$

Typical value of $\mathrm{C} \pi$ is 100 pF .

## 2. The Collector - Base Junction Capacitance $\mathbf{C} \boldsymbol{\mu}$

This is the capacitance of the reverse biased Collector - Base Junction (CBJ) .

$$
C_{\mu}=\frac{C_{\mu 0}}{\left(1+\frac{V_{C B}}{V_{0 c}}\right)^{m}}
$$

where $\mathrm{C} \mu 0$ is the value of $\mathrm{C} \mu$ at $\mathrm{VCB}=0 \mathrm{~V}$.
Voc is the CBJ built in voltage (typically 0.75 V )
m is grading coeffiecient $=1 / 2$ to $1 / 3$.
Typical value of $\mathrm{C} \mu=3 \mathrm{pF}$.


Fig. The Circuit for deriving an expression for Short Circuit current gain $\mathrm{h}_{\mathrm{fe}}(\mathrm{s})=\mathrm{Ic} / \mathrm{Ib}$.

$$
\begin{gather*}
I_{c}=\left(g_{m}-s C_{\mu}\right) V_{\pi}  \tag{1}\\
V_{\pi}=I_{b}\left(r_{\pi} / / C_{\pi} / / C_{\mu}\right)=\frac{I_{b}}{1 / r_{\pi}+s C_{\pi}+s C_{\mu}}  \tag{2}\\
h_{f e} \equiv \frac{I_{c}}{I_{b}}=\frac{g_{m}-s C_{\mu}}{1 / r_{\pi}+s\left(C_{\pi}+C_{\mu}\right)} \tag{3}
\end{gather*}
$$

Since,

$$
g_{m} \gg \omega C_{\mu}
$$

$$
\begin{aligned}
& h_{f e} \simeq \frac{g_{m} r_{\pi}}{1+s\left(C_{\pi}+C_{\mu}\right) r_{\pi}} \\
& h_{f e}=\frac{\beta_{0}}{1+s\left(C_{\pi}+C_{\mu}\right) r_{\pi}}
\end{aligned}
$$

Where $\beta_{0}$ is the low frequency value of $\beta$


Fig. Frequency Response of $h_{f e}$
$f_{\beta}$ is the 3-dB frequency

$$
\omega_{\beta}=\frac{1}{\left(C_{\pi}+C_{\mu}\right) r_{\pi}}
$$

The unity gain frequency is given by

$$
\omega_{T}=\beta_{0} \omega_{\beta}
$$

Thus

$$
\begin{gathered}
\omega_{T}=\frac{g_{m}}{C_{\pi}+C_{\mu}} \\
f_{T}=\frac{g_{m}}{2 \pi\left(C_{\pi}+C_{\mu}\right)}
\end{gathered}
$$

Frequency Response of CE Amplifier:


Low frequency Band: The gain roll off in this region is due to coupling and bypass capacitors acting as open circuit \& internal capacitances are open and in significant.

Mid frequency Band: In this region coupling and bypass capacitors will be short circuited; where as the internal capacitances are open circuits and still in effective. Hence the gain is maximum and almost constant.

High frequency Band: The gain roll off in this region is due to internal capacitances getting slowly short circuiting.

## SYLLABUS

2.1 High Frequency Response - General Consideration, The high-frequency gain function, Determining the 3-dB frequency $\mathrm{f}_{\mathrm{H}}$, Using open-circuit time constants for the approximate determination of $\mathrm{f}_{\mathrm{H}}$,

### 2.1.1 General Consideration

The various stages in an integrated- circuit cascade amplifier are directly coupled; that is they do not utilize coupling capacitors. Hence the mid band gain remains constant down to zero frequency.


### 2.1.2 The High- Frequency Gain Function:

The amplifier gain, taking into account the internal transistor capacitances, can be expressed as a function of the complex frequency variable $s$ in the general form

$$
\mathrm{A}(\mathrm{~s})=\mathrm{A}_{\mathrm{M}} \mathrm{~F}_{\mathrm{H}}(\mathrm{~S})
$$

where $\mathrm{A}_{\mathrm{M}}$ is the midband gain.
Considering the generalized transfer function

$$
F_{H}(s)=\frac{\left(1+s / \omega_{Z_{1}}\right)\left(1+s / \omega_{Z 2}\right) \ldots\left(1+s / \omega_{Z n}\right)}{\left(1+s / \omega_{P_{1}}\right)\left(1+s / \omega_{P 2}\right) \ldots\left(1+s / \omega_{P_{n}}\right)}
$$

where $\mathrm{wp} 1, \mathrm{wp} 2, \ldots . \mathrm{wpn}$ are pole frequencies.
$\mathrm{wz} 1, \mathrm{wz} 2, \ldots .2 \mathrm{zn}$ are zero frequencies.

### 2.1.3 Determining the 3-dB frequency $f_{H}$ :

i) Dominant pole is present:

The amplifier designer usually is particularly interested in the part of the high-frequencyband that is close to the midband. This is because the designer needs to estimate-and ifneed be modify-the value of the upper 3-dB frequency $\mathrm{f}_{\mathrm{H}}$. In many cases the zeros are either at infinity or suchhigh frequencies as to be of little signiicance to the determination of a) H .

If in addition oneof the poles, says $\mathrm{w}_{\mathrm{P}}$, is of much lower frequency than any of the other poles, then this polewill have the greatest effect on the value of the amplifier $w_{\mathrm{H}}$. In other words, this pole willdominate the high-frequency response of the amplifier, and the amplifier is said to have adominant-pole response. In such cases the function $\mathrm{F}_{\mathrm{H}}(\mathrm{s})$ can be approximated by

$$
F_{H}(s) \cong \frac{1}{1+s / \omega_{P_{1}}}
$$

If a dominant pole exists, then the determination of $\mathrm{W}_{\mathrm{H}}$ is greatly simplified

$$
\omega_{H} \equiv \omega_{p_{1}}
$$

This approximation is valid only when the Dominant pole is existing.

A Dominant pole is said to be existing iff the lowest frequency pole is at least two octaves ( a factor of 4) lowest from nearest pole or zero.

## ii) Dominant pole not existing

If a dominant pole does not exist, the 3-dB frequency $W_{H}$ can be determined alternatively, by an approximate formula for $\mathbf{w}_{\mathrm{H}}$, that can be derived as follows:

Consider,for simplicity, the case of a circuit having two poles and two zeros in the high-frequency
Band; that is

$$
F_{H}(s)=\frac{\left(1+s / \omega_{Z 1}\right)\left(1+\bar{s} / \omega_{Z 2}\right)}{\left(1+s / \omega_{P 1}\right)\left(1+s / \omega_{P 2}\right)}
$$

Substituting $s=j \omega$ and taking the squared magnitude gives

$$
\left|F_{H}(j \omega)\right|^{2}=\frac{\left(1+\omega^{2} / \omega_{Z 1}^{2}\right)\left(1+\omega^{2} / \omega_{Z 2}^{2}\right)}{\left(1+\omega^{2} / \omega_{P 1}^{2}\right)\left(1+\omega^{2} / \omega_{P 2}^{2}\right)}
$$

By definition, at $\omega=\omega_{H},\left|F_{H}\right|^{2}=\frac{1}{2}$; thus,

$$
\begin{aligned}
\frac{1}{2} & =\frac{\left(1+\omega_{H}^{2} / \omega_{Z 1}^{2}\right)\left(1+\omega_{H}^{2} / \omega_{Z 2}^{2}\right)}{\left(1+\omega_{H}^{2} / \omega_{P 1}^{2}\right)\left(1+\omega_{H}^{2} / \omega_{P 2}^{2}\right)} \\
& =\frac{1+\omega_{H}^{2}\left(\frac{1}{\omega_{Z 1}^{2}}+\frac{1}{\omega_{Z 2}^{2}}\right)+\omega_{H}^{4} / \omega_{21}^{2} \omega_{Z 2}^{2}}{1+\omega_{H}^{2}\left(\frac{1}{\omega_{P 1}^{2}}+\frac{1}{\omega_{P 2}^{2}}\right)+\omega_{H}^{4} / \omega_{P 1}^{2} \omega_{P 2}^{2}}
\end{aligned}
$$

Since $\mathrm{w}_{\mathrm{H}}$ is usually smaller than the frequencies of all the poles and zeros, So by neglecting the terms containing fourth order $\mathrm{w}_{\mathrm{H}}$ terms.

$$
\omega_{H} \cong 1 / \sqrt{\frac{1}{\omega_{P 1}^{2}}+\frac{1}{\omega_{P 2}^{2}}-\frac{2}{\omega_{Z 1}^{2}}-\frac{2}{\omega_{Z 2}^{2}}}
$$

This relationship can be extended to any number of poles and zeros as

$$
\omega_{H} \cong 1 / \sqrt{\left(\frac{1}{\omega_{P 1}^{2}}+\frac{1}{\omega_{P 2}^{2}}+\cdots\right)-2\left(\frac{1}{\omega_{Z 1}^{2}}+\frac{1}{\omega_{Z 2}^{2}}+\cdots\right)}
$$

## iii) Using Open Circuit Time Constants for the Approximate Determination of $\mathbf{f}_{\mathbf{H}}$

it is not a simple matterto determine the poles and zeros by quick hand analysis. In such cases an approximate value for $\mathrm{f}_{\mathrm{H}}$ can be obtained using this method.

Considering the generalized transfer function

$$
F_{H}(s)=\frac{\left(1+s / \omega_{Z 1}\right)\left(1+s / \omega_{Z 2}\right) \ldots\left(1+s / \omega_{Z_{n}}\right)}{\left(1+s / \omega_{P 1}\right)\left(1+s / \omega_{P 2}\right) \ldots\left(1+s / \omega_{P_{n}}\right)}
$$

$$
F_{H}(s)=\frac{1+a_{1} s+a_{2} s^{2}+\cdots+a_{n} s^{n}}{1+b_{1} s+b_{2} s^{2}+\cdots+b_{n} s^{n}}
$$

where the coefficients $\boldsymbol{a}$ and $\boldsymbol{b}$ are related to the frequencies of the zeros and poles, respec-tively. Specifically, the coefficient $\boldsymbol{b} \boldsymbol{1}$ is given by

$$
\begin{equation*}
b_{1}=\frac{1}{\omega_{P 1}}+\frac{1}{\omega_{P 2}}+\cdots+\frac{1}{\omega_{P n}} \tag{1}
\end{equation*}
$$

The value of $\boldsymbol{b} \mathbf{1}$ can be obtained by considering the various capacitances in the high-frequency equivalent circuit one at a time whilereducing all other capacitors to zero (or, equivalently, replacing them with open circuits). That is, to obtain the contribution of capacitance C , we reduce all other capacitances to zero, reduce input signal source to zero and determine the resistance Rio seen by Ci .

This process is repeated for all other capacitors in the circuit. The value of b 1 is computed by summing the individual time constants, hence called Open - Circuit Time Constants.

$$
b_{1}=\sum_{i=1}^{n} C_{i} R_{i o}
$$

Now the approximate value of $\mathrm{f}_{\mathrm{H}}$ can be obtained by considering there exist a dominant pole $\mathrm{w}_{\mathrm{P} 1}$ then
from eq.(1)

$$
b_{1} \simeq \frac{1}{\omega_{P_{1}}}
$$

But also upper 3dB frequency will be approximately equal to $\mathrm{w}_{\mathrm{p} 1}$ leading to the approximation

$$
\omega_{H} \simeq \frac{1}{b_{1}}=\frac{1}{\left[\sum_{i} C_{i} R_{i o}\right]}
$$

### 2.2 MILLERS THEORM:

Consider the situation in Figure 5a As a part of a larger circuit that is not shown, we have isolated two circuit nodes, labeled 1 and 2, between which an impedance Z is connected. Nodes 1 and 2 are also connected to other parts of the circuit, as signified by the broken lines emanating from the two nodes. Furthermore, it is assumed that somehow it has been determined that the node voltage at node 2 is related to the node 1 by


In typical situations $K$ is a gain factor that can be positive or negative and that has a magnitude usually larger than unity. This, however, is not an assumption for Miller's theorem. Miller's theorem states that impedance Z can be replaced by two impedances: Z 1 connected between node 1 and ground and $Z 2$ connected between node 2 and ground, where

$$
Z_{1}=Z /(1-K)
$$

and

$$
Z_{2}=Z /\left(1-\frac{1}{K}\right)
$$

The current entering in to node 1 is given as below

$$
I_{1}=\frac{V_{1}}{Z_{1}}=I=\left(\frac{V_{1}-K V_{1}}{Z}\right)
$$

similarly current entering into node 2 is given as below

$$
l_{2}=\frac{0-V_{2}}{Z_{2}}=\frac{0-K V_{1}}{Z_{2}}=l=\frac{\dot{V}_{1}-K V_{1}}{Z}
$$

Although not highlighted, the Miller equivalent circuit derived above is valid only as long as the rest of the circuit remains unchanged; otherwise the ratio of , V2 to V1 might change. It follows that the Miller equivalent circuit cannot be used directly to determine the output resistance of an amplifier. This is because in determining output resistances it is implicitly assumed that the source signal is reduced to zero and that a test-signal source (voltage or current) is applied to the output terminals-obviously a major change in the circuit, rendering the Miller equivalent circuit no longer valid.
2.3 High-frequency response of the CS and CE amplifiers : High-frequency response of the CS and CE amplifiers, Analysis using Miller's theorem, Analysis using open-circuit time constants, Exact analysis, Adapting the formulas for the case of the CE amplifier, The situation when $\mathrm{R}_{\text {sig }}$ is low

### 2.3 High-frequency response of the CS and CE amplifiers:



The above figure shows the High-frequency response equivalent circuit of the common source amplifier. This equivalent circuit applies equally well to the CE amplifier while a simple relabeling of components Cgs would be replaced by $\mathrm{C}_{\Pi}, \mathrm{Cgd}$ by $\mathrm{C} \mu$ and obviously Vgs by vп.

The input-signal source is represented by Vsig and Rsig. In some cases, however, Vsig and Rsig would be modified values of the signal-source voltage and internal resistance, taking into account other resistive components such as a bias resistor $R_{G}$ or $R_{B}$, the BJT resistances $r x$ and $r \pi$, etc. The load
resistance $R L$ represents the combination of an actual load resistance (if one is connected) and the output resistance of the current-source load. To avoid loss of gain, $R_{L}$ is usually on the same order as $r_{0}$. We combine $R_{L}$ with $r_{a}$ and denote their parallel equivalent $R_{L}{ }^{\prime}$. The load capacitance $C_{L}$ represents the total capacitance between drain (or collector) and ground; it includes the drain-to-body capacitance $C d b$ (collector-to-substrate capacitance), the input capacitance of a succeeding amplifier stage, and in some cases, as we shall see in later chapters, a deliberately introduced capacitance. In IC MOS amplifiers, $C_{L}$ can be relatively substantial.

### 2.3.1 Analysis Using Miller's Theorem

In situations when Rsig is relatively large and $C_{L}$ is relatively small, Miller's theorem can be used to obtain a quick but approximate estimate of the $3-\mathrm{dB}$ frequency $f_{H}$. Below fig shows the approximate equivalent circuit obtained for the CS case, from which we see that the amplifier has a dominant pole formed


$$
\frac{V_{e}}{V_{\mathrm{sig}}} \cong \frac{A_{2}}{1+\frac{s}{\omega_{n \prime}}}
$$

where

$$
A_{s}=-g_{n} R_{L}^{\prime}
$$

and the $3-\mathrm{dB}$ frequency $f_{H}=\omega_{H} / 2 \pi$ is given by

$$
f_{Z}=\frac{1}{2 \pi C_{\mathrm{in}} R_{\text {dg }}}
$$

where

$$
C_{\mathrm{in}}=C_{v_{s}}+C_{b \Delta}\left(1+g_{m} R_{0}^{\prime}\right)
$$

### 2.3.2 Analysis Using Open-Circuit Time Constants

The method of open-circuit time constants can be directly applied to the CS equivalent circuit of from which we see that the resistance seen by $C g s, R g s=R s i g$ and that seen by $C_{L}$ is $R_{L}^{\prime}$. The resistance $R d$ seen by $C g d$ can be found by analyzing the circuit.

$$
R_{\mathrm{kj}}=R_{\text {sie }}\left(1+g_{m} R_{L}^{\prime}\right)+R_{L}^{\prime}
$$


(a)
(b)

(c)

Thus the effective time-constant $b_{1}$ or $\tau_{H}$ can be found as

$$
\begin{aligned}
& \tau_{H}=C_{R g} R_{g s}+C_{g d} R_{g \sigma}+C_{L} R_{C_{L}} \\
& =C_{g x} R_{\mathrm{sjg}}+C_{\mathrm{zd}}\left[R_{\mathrm{sig}}\left(1+g_{\mathrm{ot}} R_{L}^{\prime}\right)+R_{L}^{\prime}\right]+C_{I} R_{L}^{\prime}
\end{aligned}
$$

and the $3 \cdot \mathrm{~dB}$ frequency $f_{H}$ is

$$
f_{H} \equiv \frac{1}{2 \pi \tau_{H I}}
$$

### 2.3.3 Exact Analysis

The approximate analysis presented above provides insight regarding the mechanism by which and the extent to which the various capacitances limit the high-frequency gain of $t$ he CS (and CE) amplifiers. A node equation at the drain provides

$$
s C_{s d}\left(V_{s s}-V_{m}\right)=g_{m} V_{s s}+\frac{V_{o}}{R_{f .}^{\prime}}+s C_{L} V_{o}
$$

which can be manipulated to the form

$$
V_{s s}=\frac{-V_{o}}{g_{0} R_{L}^{\prime}} \frac{1+s\left(C_{L}+C_{g d}\right) R_{L}^{\prime}}{1-s C_{g d} / g_{m}}
$$

A loop equation at the input yields

$$
V_{\text {sijg }}=I_{i} R_{\text {sig }}+V_{g x}
$$

in which we can substitute for $I_{i}$ from a node equation at $G$,

$$
I_{i}=s C_{y s} V_{y s}+s C_{s i i}\left(V_{g s}-V_{o}\right)
$$



$$
V_{x i t}=V_{g s}\left[1+s\left(C_{g s}+C_{s i}\right) R_{s 2 k}\right]-s C_{s_{j}} R_{s i e} V_{o}
$$

We can now substitutc in this equation for $V_{g}$ from Eq. (6.59) to obtain an equation in $V_{\omega}$ and $V_{\text {sig }}$ that can be arranged to yicld the amplifier gain as

The above transfer function indicates that the amplifier has a second-order denominator, and hence two poles. Now, since the numerator is of the first order, it follows that one of the two transmission zeros is at infinite frequency. This is readily verifiable by noting that as $s$ approaches $\infty,(\mathrm{VO} / \mathrm{V}$ sig $)$ approaches zero. The second zero is at

$$
s=s_{Z}=\frac{g_{m}}{C_{r d}}
$$

That is, it is on the positive real axis of the s-plane and has a frequency $\omega_{z}$,

$$
\omega_{z}=g_{m} / C_{g d}
$$

Since $\boldsymbol{g m}$ is usually large and $\boldsymbol{C}_{g d}$ is usually small, $\boldsymbol{f}_{z}$ is normally a very high frequency and thus has negligible effect on the value of $f_{H}$.

It is useful at this point to show a simple method for finding the value of $\boldsymbol{s}$ at which $\boldsymbol{V} \boldsymbol{o}=0$-that is, $s z$. Below Figure shows the circuit at $s=s z$. By definition, $V_{0}=0$ and a node equation at D yields


Now, since $V_{\mathrm{g},}$ is not zoro (why not?), we can divide both sides by $V_{z x}$ to obtain

$$
s_{z}=\frac{g_{m v}}{C_{d u}}
$$

Before considering the poles, we should note that as $\boldsymbol{s}$ goes toward zero, K/Vsig approaches the dc gain ( $-g_{m} R_{\mathrm{L}}^{\prime}$ ), as should be the case. Let's now take a closer look at the denominator polynomial. First, we observe that the coefficient of the $s$ term is equal to the effective time-constant $\tau_{\mathrm{H}}$ obtained using the open-circuit time-constants method. Again, this should have been expected since it is the basis for the open-circuit time-constants method. Next, denoting the frequencies of the two poles $\omega_{p 1}$ and $\omega_{p 2}$, we can express the denominator polynomial $\boldsymbol{D}(\boldsymbol{s})$ as

$$
\begin{aligned}
D(s) & =\left(1+\frac{s}{\omega_{p 1}}\right)\left(1+\frac{s}{\omega_{p p}}\right) \\
& =1+s\left(\frac{1}{\omega_{p_{1}}}+\frac{1}{\omega_{p 2}}\right)+\frac{s^{2}}{\omega_{p 1} \omega_{p_{2}}}
\end{aligned}
$$

Now, if $\omega_{p 1} \gg \omega_{p 2}$-that is, the pole at $\omega_{p 1}$ is dominant-we can approximate $\boldsymbol{D}(\boldsymbol{s})$ as

$$
D(s) \equiv 1+\frac{s}{\omega_{P 1}}+\frac{s^{2}}{\omega_{P 1} \omega_{P 2}}
$$

Equating the coefficients of the s term in denominator polynomial

$$
\begin{aligned}
& \omega_{p 1} \cong \frac{1}{\left[C_{g r}+C_{p d}\left(1+g_{m} R_{L}^{\prime}\right)\right] R_{\text {sig }}+\left(C_{L}+C_{g d}\right) R_{L}^{\prime}}
\end{aligned}
$$

### 2.3.4 Adapting the Formulas for the Case of the CE Amplifier

Adapting the formulas presented above to the case of the CE amplifier is straightforward. First, note from how Vsig and Rsigs are modified to take into account the effect of $\boldsymbol{r}_{x}$ and $\mathbf{r}_{\Pi}$

(a)

$$
\begin{aligned}
& V_{\text {sig }}^{\prime}=V_{\text {sig }} \frac{r_{\pi}}{R_{\text {sig }}+r_{x}+r_{g}} \\
& R_{\text {kig }}^{\prime}=r_{\pi} \|\left(R_{\text {sig }}+r_{x}\right)
\end{aligned}
$$


(b)

Thus the DC gain is now given by

$$
A_{M}=-\frac{r_{\pi}}{R_{\text {sig }}+r_{x}+r_{\pi}}\left(g_{m} R_{\underline{y}}^{\prime}\right)
$$

Using Millers theorem we obtain

$$
C_{\mathrm{in}}=C_{\pi}+C_{\mu}\left(1+g_{n} R_{L}^{\prime}\right)
$$

Correspondingly, the 3 - dB frequency $f_{\mathrm{H}}$ can be estimated from

$$
f_{I I} \cong \frac{1}{2 \pi C_{\text {in }} R_{\text {sig }}^{\prime}}
$$

Alternatively, using the method of open-circuit time constants yields

$$
\begin{aligned}
\tau_{H} & =C_{\pi} R_{\pi}+C_{\mu} R_{\mu}+C_{L} C_{C_{L}} \\
& =C_{\pi} R_{\mathrm{sig}}^{\prime}+C_{\mu}\left[\left(1+g_{m} R_{L}^{\prime}\right) R_{\text {sig }}^{\prime}+R_{L}^{\prime}\right]+C_{L}^{\prime} R_{L}^{\prime}
\end{aligned}
$$

from which $f_{H}$ can be estimated as

$$
f_{H} \cong \frac{1}{2 \pi \tau_{H}}
$$

The exact analysis yields the following zero frequency:

$$
f_{Z}=\frac{1}{2 \pi} \frac{g_{m}}{C_{\mu}}
$$

and, assuming that a dominant pole exists

$$
\begin{aligned}
& f_{P_{1}}=\frac{1}{2 \pi} \frac{1}{\left[C_{\pi}+C_{\mu}\left(1+g_{m 2} R_{L}^{\prime}\right)\right] R_{\mathrm{sig}}^{\prime}+\left(C_{L}+C_{\mu}\right) R_{L}^{\prime}} \\
& f_{P 2}=\frac{1}{2 \pi} \frac{\left[C_{\pi}+C_{\mu}\left(1+g_{m} R_{L}^{\prime}\right) \mid R_{\text {sig }}^{\prime}+\left(C_{L}+C_{\mu}\right) R_{L}^{\prime}\right.}{\left[C_{\pi}\left(C_{L}+C_{\mu}\right)+C_{L} C_{\mu}\right] R_{\text {sig }}^{\prime} R_{L}^{\prime}}
\end{aligned}
$$

For $f_{Z}, f_{P 2} \geqslant f_{P 1}$,

$$
f_{H} \cong f_{P 1}
$$

### 2.3.5 The Situation When $\mathrm{R}_{\text {sig }}$ Low

There are applications in which the CS amplifier is fed with a low-resistance signal source. Obviously, in such a case, the high-frequency gain will no longer be limited by the interaction of the source resistance and the input capacitance. Rather, the high-frequency limitation happens at the amplifier output, as we shall now show.
The high-frequency equivalent circuit of the common-source amplifier in the limiting case when $R_{s i g}$ is zero. The voltage transfer function $\mathrm{V}_{0} / \mathrm{V}$ sig $=V o / V g s$ can be found by setting $R_{s i g}=0$ The result is

$$
\frac{V_{o}}{V_{\text {sig }}}=\frac{\left(-g_{m} R_{L}^{\prime}\right)\left[1-s\left(C_{g d} d g_{m}\right)\right]}{1+s\left(C_{L}+C_{g d}\right) R_{L}^{\prime}}
$$

Thus, while the dc gain and the frequency of the zero do not change, the high-frequency response is now determined by a pole formed by $C L+C g d$ together with $R^{\prime} L$. Thus the $3-\mathrm{dB}$ frequency is now given by

$$
f_{H}=\frac{1}{2 \pi\left(C_{L}+C_{g d}\right)} R_{L}^{\prime}
$$

Observe that the circuit reduces to a capacitance $(C L+C g d)$ in parallel with a resistance $R_{L}^{\prime}$.
As we have seen above, the transfer-function zero is usually at a very high frequency and thus does not play a significant role in shaping the high-frequency response. The gain of the CS amplifier will therefore fall off at a rate of -6 dB /octave $(-20 \mathrm{~dB} /$ decade $)$ and reaches unity $(0 \mathrm{~dB})$ at a frequency $\mathrm{f}_{\mathrm{t}}$, which is equal to the gain - bandwidth product,

$$
\begin{aligned}
f_{t} & =\left|A_{M}\right| f_{H} \\
& =g_{m} R_{L}^{\prime} \frac{1}{2 \pi\left(C_{L}+C_{g d}\right) R_{I}^{\prime}}
\end{aligned}
$$

Thus,

$$
f_{s}=\frac{g_{\pi s}}{2 \pi\left(C_{L}+C_{K d}\right) R_{l}^{\prime}}
$$


(a)

(b)

(c)
2.4 THE CASCODE AMPLIFIER: The Cascode Amplifier, The MOS Cascode, Frequency Response of the MOS Cascode, The BIT Cascode, Darlington configuration

### 2.4 THE CASCODE AMPLIFIER

By placing a common-gate (common-base) amplifier stage in cascade with a common-source (common-emitter) amplifier stage, a very useful and versatile amplifier circuit results. It is known as the cascode configuration and has been in use for nearly three quarters of a century, obviously in a wide variety of technologies.
The basic idea behind the cascode amplifier is to combine the high input resistance and large trans conductance achieved in a common-source (common-emitter) amplifier with the current-buffering property and the superior high-frequency response of the common-gate (common-base) circuit. As will be seen shortly, the cascode amplifier can be designed to obtain a wider bandwidth but equal dc gain as compared to the common-source (common emitter) amplifier. Alternatively, it can be designed to increase the dc gain while leaving the gain-bandwidth product unchanged. Of course, there is a continuum of possibilities between these two extremes.

Although the cascode amplifier is formed by cascading two amplifier stages, in many applications it is thought of and treated as a single-stage amplifier.

### 2.4.1 The MOS Cascode


a) The MOS cascode amplifier, (b) The circuit prepared for small-signal analysis with various input and output resistances indicated, (c) The cascode with the output open-circuited.

Above Figure (a) shows the MOS cascode amplifier. Here transistor Q1 is connected in the common-source configuration and provides its output to the input terminal (i.e., source) of transistor Q2. Transistor Q2 has a constant dc voltage, V ${ }_{\text {bias, }}$ applied to its gate. Thus the signal voltage at the gate of Q 2 is zero, and $\boldsymbol{Q} 2$ is operating as a CG amplifier with a constant current load I. Obviously both Q1 and Q2 will be operating at dc drain currents equal to I.
As in previous cases, feedback in the overall circuit that incorporates the cascode amplifier establishes an appropriate dc voltage at the gate of Q1 so that its drain current is equal to I. Also, the value of V bias has to be chosen so that both Q 1 and Q 2 operate in the saturation region at all times.
Small-Signal Analysis We begin with a qualitative description of the operation of the cascode circuit. In response to the input signal voltage $\mathrm{v}_{\mathrm{i}}$ the common-source transistor Q 1 conducts a current signal $g_{\mathrm{m} 1} v_{\mathrm{i}}$ in its drain terminal and feeds it to the source terminal of the common-gate transistor Q 2 , called the cascode transistor. Transistor Q2 passes the signal current $g_{\mathrm{m} 1} v_{\mathrm{i}}$ on to its drain, where it is supplied to a load resistance $\boldsymbol{R}_{\mathrm{L}}$ at a very high output resistance, $\boldsymbol{R}$ out. The cascode transistor Q2
acts in effect as a buffer, presenting a low input resistance to $t$ he drain of Q 1 and providing a high resistance at the amplifier output.
Next we analyze the cascode amplifier circuit to determine its characteristic parameters. Toward that end Fig. (b) shows the cascode circuit prepared for small-signal analysis and with a resistance $\mathrm{R}_{\mathrm{L}}$ shown at the output. $\mathrm{R}_{\mathrm{L}}$ is assumed to include the output resistance of current source I as well as an actual load resistance, if any. The diagram also indicates various input and output resistances obtained using the results of the analysis of the CS and CG amplifiers in previous sections. Note in particular that the CS transistor Q1 provides the cascode amplifier with an infinite input resistance. Also, at the drain of Q1 looking "downward," we see the output resistance of the CS transistor Q1, $\mathrm{r}_{01}$. Looking "upward," we see the input resistance of the CG transistor Q2,

$$
R_{\mathrm{n} 2}=\frac{1}{g_{m 2}+g_{\mathrm{mb} 2}}+\frac{R_{L}}{A_{v \mathrm{v} 2}}
$$

where

$$
A_{w 2}=1+\left(g_{m 2}+g_{m b 2}\right) r_{o ?}
$$

Thus the total resistance between the drain of $Q_{1}$ and ground is

$$
R_{d 1}=r_{v 1} \|\left[\frac{1}{g_{m 2}+g_{\infty b 2}}+\frac{R_{L}}{A_{w 22}}\right]
$$

$$
\begin{gathered}
R_{o \omega 1}=r_{o 2}+A_{v 02} J_{01} \\
R_{\mathrm{sot}}=r_{o 2}+\left[1+\left(g_{m 2}+g_{m b 2}\right) r_{o 2} J r_{\Delta 1}\right. \\
R_{\mathrm{out}} \cong\left(g_{m 2} r_{o 2}\right) r_{o 1}=A_{0} r_{o 1}
\end{gathered}
$$

$$
v_{l}=v_{\text {sis }}
$$

Thus,

$$
G_{v}=A_{v}
$$

Also, note that the amplifier is unilateral; tbus,

$$
R_{e}=R_{\mathrm{cos}}
$$

$$
\frac{v_{e 1}}{v_{i}}=-g_{m 1} r_{\varphi 1}=-A_{01}
$$

$$
v_{v}=A_{v o 2} v_{v 1}
$$

Thus.

$$
\begin{aligned}
A_{00} & =-A_{01} A_{102} \\
& \vdots-A_{01} A_{02}
\end{aligned}
$$

which for the usual case of equal intrinsic gains becomes

$$
A_{\mathrm{sv}}=-\Lambda_{0}^{2}=-\left(g_{N} r_{o}\right)^{2}
$$

$$
\begin{gathered}
A_{o v}=-G_{m F} R_{0} \\
G_{m}=\frac{A_{01} A_{v o 2}}{r_{o 2}+A_{v o 2} r_{o 1}} \\
=\frac{g_{m 1} r_{o 1}\left[1+\left(g_{m 2}+g_{m b 2}\right) r_{o 2}\right]}{r_{o 2}+\left[1+\left(g_{m 22}+g_{m b 2}\right) r_{o 2}\right] r_{o 1}} \\
\equiv g_{m 1}
\end{gathered}
$$

The operation of the cascode amplifier should now be apparent: In response to $\mathrm{V}_{\mathrm{i}}$ the CS transistor provides a drain current $\mathrm{gm}_{1} \mathrm{~V}_{\mathrm{i}}$, which the CG transistor passes on to $R_{\mathrm{L}}$ and, in the process, increases the output resistance by $\mathrm{A}_{0}$. It is the increase in Rout to $\mathrm{A}_{0} \mathrm{r}_{0}$ that increases the open-circuit voltage gain to $(\mathrm{gm})\left(\mathrm{A}_{0} \mathrm{ro}\right)=\mathrm{A}_{0}{ }^{2}$

(a)
(b)

(c)

From the above circuits

$$
A_{0}=-A_{0}^{2} \frac{R_{i}}{R_{L}+A_{0} r_{o}}
$$

$$
\begin{aligned}
R_{L}=A_{0} r_{o}, \bar{A}_{\psi} & =-A_{0}^{2} / 2 . \\
\frac{u_{o l}}{v_{j}} & =-g_{x n}\left[r_{o} \|\left(\frac{1}{g_{a v}}+\frac{R_{L}}{A_{0}}\right)\right]
\end{aligned}
$$

For $R_{L}=A_{0} r_{\nu,}$,

$$
\left.\begin{array}{rl}
\frac{v_{a 1}}{v_{\mathrm{i}}} & =-g_{o v}\left[r_{o} \|\left(\frac{1}{g_{m}}+r_{o}\right)\right] \\
& \cong-\frac{1}{2} g_{m} r_{v}=-\frac{1}{2} A_{01} \\
\frac{v_{o 1}}{v_{i}} & =-g_{m}\left[r_{o} \|\left(\frac{1}{g_{o}}+\frac{1}{g_{m}}\right)\right] \\
\equiv-2 \mathrm{~V} / \mathrm{V}
\end{array}\right] \begin{aligned}
& A_{v}=-A_{0}^{2} \frac{r_{o}}{r_{o}+A_{0} r_{o}} \equiv-A_{0}
\end{aligned}
$$

### 2.4.2 Frequency Response of the MOS Cascode



Above Figure shows the cascode amplifier with all transistor internal capacitances indicated. Also included is a capacitance $\mathrm{C}_{\mathrm{L}}$ at the output node to represent the combination of $\mathrm{C}_{\mathrm{db} 2}$, the input capacitance of a succeeding amplifier stage (if any), and a load capacitance (if any). Note that $\mathrm{C}_{\mathrm{db}}$ and $\mathrm{C}_{\mathrm{g} s 2}$ appear in parallel, and we shall combine them in the following analysis. Similarly, $C_{\mathrm{L}}$ and $\mathrm{C}_{\mathrm{gd} 2}$ appear in parallel and will be combined.

1. Capacitance $\mathrm{C}_{\mathrm{gs} 1}$ sees a resistance $\mathrm{R}_{\text {sig }}$.
2. Capacitance $\mathrm{C}_{\mathrm{gdl}}$ sees a resistance $\mathrm{Rgdl}^{\text {g }}$, which can be obtained by adapting the formula

$$
R_{g d 1}=\left(1+g_{m 1} R_{d 1}\right) R_{\mathrm{sig}}+R_{d 1}
$$

3.Capacitance ( $\mathrm{C}_{\mathrm{dbl}}+C_{\mathrm{g} s 2}$ ) sees a resistance $R_{\mathrm{dl}}$.
4. Capacitance ( $C_{\mathrm{L}}+R_{\mathrm{gd} 2}$ ) sees a resistance ( $R_{\mathrm{L}} \backslash \backslash$ out).

With the resistances determined, the effective time constant $\tau_{\mathrm{H}}$ can be computed as

$$
\begin{aligned}
\tau_{H}= & C_{g s 1} R_{\mathrm{sig}}+C_{g d 1}\left[\left(1+g_{\mathrm{in1}} R_{d i}\right) R_{\mathrm{sig}}+R_{d \mathrm{~d}}\right] \\
& +\left(C_{d b 1}+C_{g 22}\right) R_{d 1}+\left(C_{L}+C_{g d 2}\right)\left(R_{L} \| R_{\mathrm{cut}}\right)
\end{aligned}
$$

and the $3-\mathrm{dB}$ frequency $f_{H}$ as

$$
f_{H} \cong \frac{1}{2 \pi \tau_{H}}
$$

To gain insight regarding what limits the high-frequency gain of the MOS cascode amplifier, we rewrite above equation in the form

$$
\begin{aligned}
& \tau_{H}=R_{s i 2}\left[C_{g s 1}+C_{g d 1}\left(1+g_{m 1} R_{d 1}\right)\right]+R_{d i 1}\left(C_{g d 1}+C_{d b 1}+C_{g ; 2}\right) \\
& +\left(R_{L} \| R_{\text {cut }}\right)\left(C_{L}+C_{g d 2}\right)
\end{aligned}
$$

In the case of a large Rsig, the first term can dominate, especially if the Miller multiplier $\left(1+\mathrm{g}_{\mathrm{m} 1} \mathrm{R}_{\mathrm{d} 1}\right)$ is large. This in turn happens when the load resistance $R_{\mathrm{L}}$ is large (on the order of $\mathrm{A}_{0} \mathrm{r}_{\mathrm{o}}$ ), causing $R \mathrm{in} 2$ to be large and requiring the first stage, Q 1 to provide a large proportion of the gain. It follows that when jRs i g is large, to extend the bandwidth we have to lower $R L$ to the order of $r 0$. This in turn lowers $R m 2$ and hence $R d i$ and renders the Miller effect insignificant. Note, however, that the dc gain of the cascode will then be A 0 . Thus, while the
dc gain will be the same as (or a little higher than) that achieved in a CS amplifier, the bandwidth will be greater.

$$
\tau_{H} \cong\left(C_{L}+C_{R^{\prime 2} 2}\right)\left(R_{L} \| R_{\mathrm{ou}}\right)
$$

and the $3-\mathrm{dB}$ frequency becomes

$$
\begin{array}{r}
f_{H}=\frac{1}{2 \pi\left(C_{L}+C_{\mathrm{gd} d}\right)\left(R_{L} \| R_{\text {out }}\right)} \\
f_{\mathrm{t}} \cong \frac{1}{2 \pi} \frac{3_{m}}{C_{L}+C_{\mathrm{g} d 2}}
\end{array}
$$




### 2.4.2 The BJT Cascode

Below Figure shows the BJT cascode amplifier. The circuit is very similar to the MOS cascode, and the small-signal analysis follows in a similar fashion, as indicated in Fig. (b). Here we have shown the various input and output resistances. Observe that unlike the MOSFET cascode, which has an infinite input resistance, the B JT cascode has an input resistance of $\mathrm{r}_{\Pi_{1}}$ (neglecting $\mathrm{r}_{\mathrm{x}}$ ). The formula for $\mathrm{R}_{\mathrm{in} 2}$ is the one we found in the analysis of the common-base circuit The output resistance Rout= $\beta_{2} r_{02}$ is found by substituting $\operatorname{Re}=r_{01}$ and making the approximation that $g_{m} r_{0} \gg$ $\beta$. Recall that $\beta \mathrm{r}_{0}$ is the largest output resistance that a CB transistor can provide.

The open-circuit voltage gain Avo and the no-load input resistance Ri can be found from the circuit in Fig. (c), in which the output is open-circuited. Observe that Rin2 $=r_{\Pi 2}$, which is
usually much smaller than $\mathrm{r}_{\mathrm{o} 1}$. As a result the total resistance between the collector of Q1 and ground is approximately $r_{\Pi 2}$; thus the voltage gain realized in the CE transistor Q1 is - $\mathrm{g}_{\mathrm{m}}$ $r_{\Pi 2}=\beta \cdot$ Recalling that the open-circuit voltage gain of a CB amplifier is $\left(1+g_{\mathrm{m}} r_{0}\right)=A 0$, we see that the voltage gain $\mathrm{A}_{\mathrm{v} 0}$ is

$$
A_{\infty 0}=-\beta A_{0}
$$


(a)

(b)

(c)

(a)

(b)

(c)

Putting all of these results together we obtain for the BJT cascode amplifier the equivalent circuit shown in below Fig (a)


We note that compared to the common-emitter amplifier, cascoding increases both the open circuit voltage gain and the output resistance by a factor equal to the transistor $\beta$. This should be contrasted with the factor $\mathrm{A}_{0}$ encountered in the MOS cascode.
The equivalent circuit can be easily converted to the transconductance form shown in Fig. (b). It shows that the short-circuit transconductance $g m$ of the cascode amplifier is equal to the transconductance $g m$ of the BJTs. This should have been expected since Q 1 provides a current $\mathrm{g}_{\mathrm{m} 1} \mathrm{~V}_{\mathrm{i}}$ to the emitter of the cascode transistor Q2, which in turn passes the current on (assuming $a_{2}=1$ ) to its collector and to the load resistance $R_{\mathrm{L}}$. In the process the cascode transistor raises the resistance level from $r_{0}$ at the collector of Q 1 to $\beta_{\mathrm{r} 0}$ at the collector of $Q 2$. This is the by-now-familiar currentbuffering action of the common-base transistor.
The voltage gain of the CE transistor $Q x$ can be determined from the equivalent circuit in Fig. (c). The resistance between the collector of Q1 and ground is the parallel equivalent of the output resistance of Q1, $r 0$, and the input resistance of the CB transistor, Q2, namely $\mathrm{R}_{\mathrm{in} 2}$. Note that for $\mathrm{R}_{\mathrm{L}}<\mathrm{r}_{0}$ the latter reduces to re, as expected. However, $\mathrm{R}_{\mathrm{in} 2}$ increases as $\mathrm{R}_{\mathrm{L}}$ is increased. Of particular interest is the value of $\mathrm{R}_{\mathrm{in} 2}$ obtained for $R_{\mathrm{L}}=B_{\mathrm{r} 0}$, namely $\mathrm{Rin} 2=\mathrm{r} \quad \Pi / 2$. It follows that for this value of $R_{\mathrm{L}}$ the CE stage has a voltage gain of $-\beta / 2$.

### 2.5 The Darlington Configuration

Below Figure shows a popular BJT circuit known as the Darlington configuration. It can be thought of as a variation of the CC - CE circuit with the collector of Q1 connected to that of Q2. Alternatively, the Darlington pair can be thought of as a composite transistor with $\boldsymbol{\beta}_{\boldsymbol{=}} \boldsymbol{\beta}_{1} \boldsymbol{\beta}_{2}$. It can therefore be used to implement a high-performance voltage follower, as illustrated in Fig. (b). Note that in this application the circuit can be considered as the cascade connection of two commoncollector transistors (i.e., a CC - C C configuration).


## UNIT - III

## DIFFERENTIAL AMPLIFIERS

## Objectives

- To illustrate MOS and BJT differential amplifierswith common - mode input and differential mode inputs.
- To analyze large-signal and small-signal operation of MOS and BJT differential amplifiers.


## Syllabus

MOS differential pair - operation with a common-mode input voltage, operation with a differential input voltage, Large -signal operation, small-signal operation of the MOS differential pair, differential gain, common-mode gain and common-mode rejection ratio (CMRR), frequency-response of resistively loaded MOS differential amplifier, The BJT differential pair - Basic operation, large-signal operation, small-signal operation

## Pre-requisites

Studentshould have prior knowledge offundamental circuit analysistechniques and basic electronics background of BJT, MOSFET

## Outcomes

Student will be able to

- analyze the MOS and BJT differential pair circuits.
- find the differential gain, common-mode gain, and CMRR of a differential amplifier.
- analyze the frequency response of a differential amplifier.


## Introduction

- The differential-pair or differential-amplifier configuration is the most widely used buildingblock in analog integrated-circuit design.
- For instance, the input stage of every op amp is a differential amplifier. Also, the BJT differential amplifier is the basis of a very-high-speed logic circuit family.
- There are two reasons why differential amplifiers are so well suited for IC fabrication:
$\checkmark$ The performance of the differential pair depends critically on the matching between the two sides of the circuit. Integrated-circuit fabrication is capable of providing matched devices whose parameters track over wide ranges of changes in environmental conditions.
$\checkmark$ Differential amplifiers utilize more components (approaching twice as many) than single-ended circuits. The advantage of integrated-circuit technology is the availability of large numbers of transistors at relatively low cost.


## Why the differential amplifiers are preferred over single-ended amplifiers?

Basically, there are two reasons for using differential in preference to single-ended amplifiers.

1. Differential circuits are much less sensitive to noise and interference thansingleended circuits.

Consider two wires carrying a small differentialsignal as the voltage difference between the two wires. Now, assume that there is an interferencesignal that is coupled to the two wires, either capacitively or inductively. As the twowires are physically close together, the interference voltages on the two wires (i.e., betweeneach of the two wires and ground) will be equal. Since, in a differential system, only the differencesignal between the two wires is sensed, it will contain no interference component!
2. For preferring differential amplifiers is that the differential configurationenables us to bias the amplifier and to couple amplifier stages together without the needfor bypass and coupling capacitors such as those utilized in the design of discretecircuit amplifiers.

This is another reason why differential circuits are ideallysuited for IC fabrication where large capacitors are impossible to fabricate economically.

### 3.1 The MOS differential pair

Figure 3.1 shows the basic MOS differential-pair configuration. It consists of two matchedtransistors, $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$, whose sources are joined- together and biased by a constant-current source I.

- Assume that the current source is ideal and that it has infinite output resistance.
- Although each drain is shown connected to the positive supply through a resistance $\mathrm{R}_{\mathrm{D}}$, in most cases active (current-source) loads are employed.
- Whatever type of load is used, it is essential that the MOSFETs not enter the triode region of operation.


Fig.3.1: The basic MOS differential - pair configuration

### 3.1.1 Operation with a Common-Mode Input Voltage

- To see how the differential pair works, consider first the case of the two gate terminalsjoined together and connected to a voltage $v_{\mathrm{CM}}$, called the common-mode voltage.

That is, as shown in Fig. 3.2, $v_{\mathrm{G} 1}=v_{\mathrm{G} 2}=v_{\mathrm{CM}}$. Since $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are matched, it follows from symmetry that the current / will divide equally between the two transistors. Thus, $i_{D 1}=i_{D 2}=I / 2$, and the voltage at the sources,

$$
\begin{equation*}
\mathrm{V}_{\mathrm{S}}=\mathrm{V}_{\mathrm{CM}}-\mathrm{V}_{\mathrm{GS}} . \tag{1}
\end{equation*}
$$

where $\mathrm{V}_{\mathrm{GS}}$ is the gate-to-source voltage corresponding to a drain current of $1 / 2$. Neglectingchannel-length modulation, $\mathrm{V}_{\mathrm{GS}}$ and I/2 are related by

$$
\begin{equation*}
\frac{I}{2}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L}\left(V_{G S}-V_{t}\right)^{2} \tag{2}
\end{equation*}
$$

or in terms of the overdrive voltage $\mathrm{V}_{\mathrm{ov}}$,

$$
\begin{align*}
V_{O V} & =V_{G S}-V_{t} \\
\frac{I}{2} & =\frac{1}{2} k_{n}^{\prime} \frac{W}{L} V_{o V}^{2} \\
V_{O V} & =\sqrt{I / k_{n}^{\prime}(W / L)} . \tag{3}
\end{align*}
$$

The voltage at each drain will be

$$
\begin{equation*}
v_{D 1}=v_{D 2}=V_{D D}-\frac{I}{2} R_{D} \tag{4}
\end{equation*}
$$

Thus, the difference in voltage between the two drains will be zero.

- If the value of the common-mode voltage $v_{\mathrm{CM}}$ is varied, as long as $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ remain in the saturation region, the current I will divide equally between $Q_{1}$ and $Q_{2}$ and the voltages at the drains will not change. Thus the differential pair does not respond to (i.e., it rejects) common-mode input signals.
- An important specification of a differential amplifier is its input common-mode range. This is the range of $v_{\mathrm{CM}}$ over which the differential pair operates properly.


Fig.3.2: The MOS differential pair with a common-mode input voltage, $v_{\mathrm{CM}}$

- The highest value of $v_{\mathrm{CM}}$ is limited by the requirement that $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ remain in saturation,

$$
\begin{equation*}
V_{C M \max }=V_{t}+V_{D D}-\frac{I}{2} R_{D} \tag{5}
\end{equation*}
$$

- The lowest value of $v_{\mathrm{Cm}}$ is determined by the need to allow for a sufficient voltage across current source / for it to operate properly. If a voltage $\mathrm{V}_{\mathrm{CS}}$ is needed across the current source, then

$$
\begin{equation*}
V_{C M \min }=-V_{S S}+V_{C S}+V_{t}+V_{O V} \tag{6}
\end{equation*}
$$

### 3.1.2 Operation with a Differential Input Voltage

If we apply a difference or differential input voltage by grounding the gate of $\mathrm{Q}_{2}$ (i.e., setting $v_{\mathrm{G} 2}=0$ ) and applying a signal $v_{\mathrm{id}}$ to the gate of $\mathrm{Q}_{1}$ as shown in Fig. 3.3. Since $v_{\mathrm{id}}=v_{\mathrm{GS} 1}-v_{\mathrm{GS} 2}$.


Fig.3.3: The MOS differential pair with a differential input signal $v_{\mathrm{id}}$ applied.

- With $v_{\text {id }}$ positive: $v_{\mathrm{GS} 1}>v_{\mathrm{GS} 2}, i_{\mathrm{D} 1}>i_{\mathrm{D} 2}$ and $v_{\mathrm{D} 1}<v_{\mathrm{D} 2}$, thus $\left(v_{\mathrm{D} 2}-v_{\mathrm{D} 1}\right)$ will be positive.
- With $v_{\mathrm{id}}$ negative: $V_{\mathrm{GS} 1}<v_{\mathrm{GS} 2}, i_{\mathrm{D} 1}<i_{\mathrm{D} 2}$, and $V_{\mathrm{D} 1}>v_{\mathrm{D} 2}$; thus $\left(v_{\mathrm{D} 2}-V_{\mathrm{D} 1}\right)$ will be negative.
- From the above, we see that the differential pair responds to difference-mode or differentialinput signals by providing a corresponding differential output signal between the twodrains.
- At this point, it is useful to inquire about the value of $v_{\text {id }}$ that causes the entire biascurrent / to flow in one of the two transistors.
- In the positive direction, this happens when $v_{\mathrm{GS} 1}$ reaches the value that corresponds to $i_{\mathrm{D} 1}=\mathrm{I}$, and $v_{\mathrm{GS} 2}$ is reduced to a value equal to thethreshold voltage $\mathrm{V}_{\mathrm{t}}$, at which point $v_{\mathrm{S}}=-\mathrm{V}_{\mathrm{t}}$. The value of $v_{\mathrm{GS} 1}$ can be found from

$$
\begin{align*}
& \begin{array}{l}
I=\frac{1}{2}\left(k_{n}^{\prime} \frac{W}{L}\right)\left(v_{G S 1}-V_{t}\right)^{2} \ldots \\
{ }^{v_{G S 1}}
\end{array}  \tag{7}\\
& =V_{t}+\sqrt{2 I / k_{n}^{\prime}(W / L)} \\
& \quad=V_{t}+\sqrt{2} V_{O V} \tag{8}
\end{align*}
$$

where $V_{\text {ov }}$ is the overdrive voltage corresponding to a drain current of $\mathrm{I} / 2$.

- Thus, the value of $v_{\text {id }}$ at which the entire bias current $I$ is steered into $\mathrm{Q}_{1}$ is

$$
\begin{align*}
v_{i d \max } & =v_{G S 1}+v_{S} \\
& =V_{t}+\sqrt{2} V_{O V}-V_{t} \\
& =\sqrt{2} V_{O V} \tag{9}
\end{align*} .
$$

- If $v_{\mathrm{id}}$ is increased beyond $\sqrt{2} \mathrm{~V}_{\mathrm{ov}}$, $i_{\mathrm{D} 1}$ remains equal to $\mathrm{I}, v_{\mathrm{GS} 1}$ remains equal to $\left(\mathrm{V}_{\mathrm{t}}+\right.$ $\sqrt{ } 2 \mathrm{~V}_{\mathrm{ov}}$ ), and $v_{\mathrm{s}}$ rises correspondingly, thus keeping $\mathrm{Q}_{2}$ off.
- In a similar manner we can show that in the negative direction, as $v_{\text {id }}$ reaches $\sqrt{ } 2 \mathrm{~V}_{\text {ov }}, \mathrm{Q}_{1}$ turns off and $\mathrm{Q}_{2}$ conducts the entire bias current I .
- Thus the current / can be steered from one transistor to the other by varying $v_{\mathrm{id}}$ in the range

$$
\begin{equation*}
-\sqrt{2} V_{O V} \leq v_{i d} \leq \sqrt{2} V_{O V} \tag{10}
\end{equation*}
$$

$\qquad$
which defines the range of differential-mode operation. Finally, observe that we have assumed that $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ remain in saturation even when one of them is conducting the entire current I.

### 3.1.3 Large-Signal Operation

- We derive the expressions for the drain currents $i_{D 1}$ and $i_{D 2}$ in terms of the input differential signal $v_{\mathrm{id}}=v_{\mathrm{G} 1}-v_{\mathrm{G} 2}$.
- Assume that the circuit maintains $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ out of the triode region of operation at all times.


Fig.3.4: The MOSFET differential pair for the purpose of deriving the transfer characteristics, $i_{\mathrm{D} 1}$ and $i_{\mathrm{D} 2}$ versus $v_{\mathrm{id}}=v_{\mathrm{G} 1}-v_{\mathrm{G} 2}$.

- The following derivation assumes that the differential pair is perfectly matched and neglects channel-length modulation $(\lambda=0)$ and the body effect.
- To begin with, we express the drain currents of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ as

$$
\begin{align*}
& i_{D 1}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L}\left(v_{G S 1}-V_{t}\right)^{2}  \tag{11}\\
& i_{D 2}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L}\left(v_{G S 2}-V_{t}\right)^{2} \tag{12}
\end{align*}
$$

Taking the square roots of both sides of each of the above equations, we get

$$
\begin{equation*}
\sqrt{i_{D 1}}=\sqrt{\frac{1}{2} k_{n}^{\prime} \frac{W}{L}}\left(v_{G S 1}-V_{t}\right) \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
\sqrt{i_{D 2}}=\sqrt{\frac{1}{2} k_{n}^{\prime} \frac{W}{L}}\left(v_{G S 2}-V_{t}\right) . \tag{14}
\end{equation*}
$$

Subtracting Eq. 14 from Eq. 13 and substituting

$$
\begin{align*}
& v_{G S 1}-v_{G S 2}=v_{G 1}-v_{G 2}=v_{i d} .  \tag{15}\\
& \sqrt{i_{D 1}}-\sqrt{i_{D 2}}=\sqrt{\frac{1}{2} k_{n}^{\prime} \frac{W}{L} v_{i d}} . \tag{16}
\end{align*}
$$

The constant-current bias imposes the constraint Squaring both sides of Eq. 16 and substituting for $i_{D 1}+i_{D 2}=I$ gives

$$
\begin{equation*}
2 \sqrt{i_{D 1} i_{D 2}}=I-\frac{1}{2} k_{n}^{\prime} \frac{W}{L} v_{i d}^{2} . \tag{17}
\end{equation*}
$$

Substituting for $i_{D 2}$ from Eq. 17 as $i_{D 2}=I-i_{D 1}$ and squaring both sides of the resulting equation provides a quadratic equation in $i_{D 1}$ that can be solved to yield

$$
\begin{equation*}
i_{D 1}=\frac{I}{2} \pm \sqrt{k_{n}^{\prime} \frac{W}{L} I}\left(\frac{v_{i d}}{2}\right) \sqrt{1-\frac{\left(v_{i d} / 2\right)^{2}}{I / k_{n}^{\prime} \frac{W}{L}}} \tag{18}
\end{equation*}
$$

Now, since the increment in $i_{D 1}$ above the bias value of I/2 must have the same polarity as $v_{\text {id }}$, only the root with the " + " sign in the second term is physically meaningful; thus,

$$
\begin{equation*}
i_{D 1}=\frac{I}{2}+\sqrt{k_{n}^{\prime} \frac{W}{L} I}\left(\frac{v_{i d}}{2}\right) \sqrt{1-\frac{\left(v_{i d} / 2\right)^{2}}{I / k_{n}^{\prime} \frac{W}{L}}} \tag{19}
\end{equation*}
$$

The corresponding value of $i_{D 2}$ is found from $i_{D 2}=I-i_{D 1}$ as

$$
\begin{equation*}
i_{D 2}=\frac{I}{2}-\sqrt{k_{n}^{\prime} \frac{W}{L} I}\left(\frac{v_{i d}}{2}\right) \sqrt{1-\frac{\left(v_{i d} / 2\right)^{2}}{I / k_{n}^{\prime} \frac{W}{L}}} \tag{20}
\end{equation*}
$$

At the bias (quiescent) point, $v_{\mathrm{id}}=0$, leading to

$$
\begin{equation*}
i_{D 1}=i_{D 2}=\frac{I}{2} \tag{21}
\end{equation*}
$$

Correspondingly,

$$
\begin{equation*}
v_{G S 1}=v_{G S 2}=V_{G S} . \tag{22}
\end{equation*}
$$

Where

$$
\begin{equation*}
\frac{I}{2}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L}\left(V_{G S}-V_{t}\right)^{2}=\frac{1}{2} k_{n}^{\prime} \frac{W}{L} V_{O V}^{2} \tag{23}
\end{equation*}
$$

Now the expressions for $i_{D 1}$ and $i_{D 2}$ in the alternative form is given as

$$
\begin{gather*}
i_{D 1}=\frac{I}{2}+\left(\frac{I}{V_{O V}}\right)\left(\frac{v_{i d}}{2}\right) \sqrt{1-\left(\frac{v_{i d} / 2}{V_{O V}}\right)^{2}} .  \tag{24}\\
i_{D 2}=\frac{I}{2}-\left(\frac{I}{V_{O V}}\right)\left(\frac{v_{i d}}{2}\right) \sqrt{1-\left(\frac{v_{i d} / 2}{V_{O V}}\right)^{2}} . \tag{25}
\end{gather*}
$$

These two equations describe the effect of applying a differential input signal $v_{\mathrm{id}}$ on the currents $i_{D 1}$ and $i_{D 2}$.

- At $v_{\mathrm{id}}=0$, the two currents are equal to $\mathrm{I} / 2$. Making $v_{\mathrm{id}}$ positive causes $i_{D 1}$ to increase and $i_{D 2}$ to decrease by equal amounts, to keep the sum constant, $i_{D 1}+i_{D 2}=I$. The current is steered entirely into $Q 1$ when $v_{\text {id }}$ reaches the value $\sqrt{2} V_{O V}$.


Fig.3.5:Normalized plots of the currents in a MOSFET differential pair. Note that Vov is the overdrive voltage at which $Q_{1}$ and $Q_{2}$ operate when conducting drain currents equal to I/2, the equilibrium situation. Note that these graphs are universal and apply to any MOS differential pair.

- For $v_{\text {id }}$ negative, identical statements can be made by interchanging $i_{D 1}$ and $i_{D 2}$. In this case, ${ }^{v_{i d}}=-\sqrt{2} V_{O V}$ steers the current entirely into $Q_{2}$. Finally, note that the plots in Fig. 3.5 are universal, as they apply to any MOS differential pair.
- The transfer characteristics of Eqs. (24)and(25) and Fig. 3.6 are obviously nonlinear. This is due to the term involving $\mathrm{V}_{\mathrm{id}^{2}}$. Since we are interested in obtaining linear amplification from the differential pair, we will strive to make this term as small as
possible. For a given value of $V_{o v}$, the only thing we can do is keep $V_{i d} / 2$ much smaller than $V_{\text {ov }}$, which is the condition for the small-signal approximation. It results in

$$
\begin{equation*}
i_{D 1} \simeq \frac{I}{2}+\left(\frac{I}{V_{O V}}\right)\left(\frac{v_{i d}}{2}\right) \tag{26}
\end{equation*}
$$

And

$$
\begin{equation*}
i_{D 2} \simeq \frac{I}{2}-\left(\frac{I}{V_{O V}}\right)\left(\frac{v_{i d}}{2}\right) \tag{27}
\end{equation*}
$$

which, as expected, indicate that $i_{D 1}$ increases by an increment $i_{\mathrm{d}}$, and $i_{D 2}$ decreases by the same amount, $i_{\mathrm{d}}$, where $i_{\mathrm{d}}$ is proportional to the differential input signal $v_{\mathrm{id}}$,

$$
\begin{equation*}
i_{d}=\left(\frac{I}{V_{O V}}\right)\left(\frac{v_{i d}}{2}\right) \tag{28}
\end{equation*}
$$



Fig.3.6: The linear range of operation of the MOS differential pair can be extended by operating the transistor at a higher value of Vov.

### 3.2 Small-Signal Operation of the MOS Differential Pair

### 3.2.1 Differential Gain

Figure 3.7 shows the MOS differential amplifier with input voltages

$$
\begin{aligned}
& \quad v_{G 1}=V_{C M}+\frac{1}{2} v_{i d} \text { and } \\
& v_{G 2}=V_{C M}-\frac{1}{2} v_{i d}
\end{aligned}
$$


(a)


Fig.3.7: Small-signal analysis of the MOS differential amplifier. (a) The circuit with a common-mode voltage applied to set the dc bias voltage at the gates and with $v_{i d}$ applied in a complementary (or balanced) manner. (b) The circuit prepared for small-signal analysis. (c) An alternative way of looking at the small signal operation of the circuit.
 of the differential amplifier. It is needed in order to set the dc voltage of the MOSFET gates.Typically $V_{\text {смis }}$ at the middle value of the power supply. Thus, for our case, where two complementary supplies are utilized, $V_{\text {Смі }}$ typically 0 V .

- The differential input signal $v_{\mathrm{id}}$ is applied in a complementary (or balanced) manner; that is, $v_{G 1}$ is increased by $v_{\mathrm{id}} / 2$ and $v_{\mathrm{G} 2}$ is decreased by $v_{\mathrm{id}} / 2$.
- If the differential amplifier were fed from the output of another differential-amplifier stage.
- As indicated in above Fig.3.7, the amplifier output can be taken either between one of the drains and ground or between the two drains.
- In the first case, the resulting single-ended outputs $v_{o 1}$ and $v_{02}$ will be riding on top of the dc voltages at the drains, $\mathrm{V}_{\mathrm{DD}}-\mathrm{I} / 2 \mathrm{R}_{\mathrm{D}}$.
- This is not the case when the output is taken between the two drains; the resulting differential output $v_{\mathrm{od}}$ (having a $0-\mathrm{V}$ dc component) will be entirely a signal component.
- To analyze the small-signal operation of the differential amplifier of Fig.3.7(a), and to determine its voltage gain in response to the differential input signal $v_{\mathrm{id}}$.
- From the Fig.3.7 (b), the circuit with the power supplies grounded, the bias current source I removed, and $V_{\text {сме }}$ liminated; that is, only signal quantities are indicated.
- We neglect the effect of the MOSFET $r_{0}$. Finally note that each of $Q_{1}$ and $Q_{2}$ is biased at a dc current of $I / 2$ and is operating at an overdrive voltage $V_{\text {ov. }}$
- From the symmetry of the circuit and because of the balanced manner in which $v_{\mathrm{id} i}$ is applied, we observe that the signal voltage at the joint source connection must be zero, acting as a sort of virtual ground. Thus $Q_{1}$ has a gate-to-source voltage signal $v_{g s 1}=\mathrm{V}_{\mathrm{id}} / 2$ and $Q_{2}$ has $v_{g s 2}=-\mathrm{V}_{\mathrm{id}} / 2$.
- Assuming ${ }^{v_{i d}} / 2 \leftrightarrow V_{O V}$, the condition for the small-signal approximation, the changes resulting in the drain currents of $Q_{1}$ and $Q_{2}$ will be proportional to $v_{g s 1}$ and $v_{g s 2}$, respectively.
- Thus $Q_{1}$ will have a drain current increment $g_{\mathrm{m}}\left(\mathrm{V}_{\mathrm{id}} / 2\right)$ and $Q 2$ will have a drain current decrement $g_{\mathrm{m}}\left(\mathrm{V}_{\mathrm{id}} / 2\right)$, where $g_{\mathrm{m}}$ denotes the equal transconductances of the two devices,

$$
\begin{equation*}
g_{m}=\frac{2 I_{D}}{V_{O V}}=\frac{2(I / 2)}{V_{o V}}=\frac{I}{V_{o V}} \tag{29}
\end{equation*}
$$

The output voltages for two individual stages are given as

$$
\begin{align*}
& v_{o 1}=-g_{m} \frac{v_{i d}}{2} R_{D}  \tag{30}\\
& v_{o 2}=+g_{m} \frac{v_{i d}}{2} R_{D} \tag{31}
\end{align*}
$$

If the output is taken in a single-ended fashion, the resulting gain becomes

$$
\begin{align*}
& \frac{v_{o 1}}{v_{i d}}=-\frac{1}{2} g_{m} R_{D}  \tag{32}\\
& \frac{v_{o 2}}{v_{i d}}=\frac{1}{2} g_{m} R_{D} . \tag{33}
\end{align*}
$$

Alternatively, if the output is taken differentially, the gain becomes

$$
\begin{equation*}
A_{d} \equiv \frac{v_{o d}}{v_{i d}}=\frac{v_{o 2}-v_{o 1}}{v_{i d}}=g_{m} R_{D} . \tag{34}
\end{equation*}
$$

### 3.2.2 Common-Mode Gain and Common-Mode Rejection Ratio (CMRR)

- Figure 3.8 (a) shows a MOS differential amplifier biased with a current source having an output resistance $\mathrm{R}_{\mathrm{ss}}$.
- The dc voltage at the input is defined by $\mathrm{V}_{\text {см }}$. Here, however, we also have an incremental signal $\mathrm{V}_{\mathrm{icm}}$ applied to both input terminals. This common mode input signal can represent an interference signal or noise that is picked up by both inputs and is clearly undesirable.
- To find how much of $\mathrm{V}_{\mathrm{icm}}$ makes its way to the output of the amplifier, we determine the common-mode gain of the amplifier, analyze the effect of $\mathrm{R}_{\mathrm{ss}}$ on the bias current of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$. That is, with set to zero, the bias current in each of and will no longer be $I / 2$ but will be larger than $I / 2$ by an amount determined by $\mathrm{V}_{\mathrm{CM}}$ and $\mathrm{R}_{\mathrm{SS}}$.
- However, since $R_{\text {SS }}$ is usually very large, this additional dc current in each of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ is usually small and we shall neglect it, thus assuming that and continue to operate at a bias current of $I / 2$.

(a)

(b)


Fig.3.8: (a) A MOS differential amplifier with a common-mode input signal $v_{i c m}$ superimposed on the input dc commonmode voltage $V_{C M}$. (b) The amplifier circuit prepared for small-signal analysis. (c) The amplifier circuit with the transistors replaced with their T model and $r_{o}$ neglected. (d) The circuit in (b) split into its two halves; each half is called the "CM halfcircuit."

- To determine the response of the differential amplifier to the common-mode input signal, consider the circuit in Fig.3.8, where we have replaced each of $V_{D D}$ and $V_{S S}$ by a short circuit and $I$ by an open circuit.
- The circuit is obviously symmetrical, and thus the two transistors will carry equal signal currents, denoted $i$. The value of $I$ can be easily determined by replacing each of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ with its T model and, for simplicity, neglecting $\mathrm{r}_{0}$. The resulting equivalent circuit is shown in Fig. 3.8(c), from which we can write

$$
\begin{equation*}
v_{i c m}=\frac{i}{g_{m}}+2 i R_{S S} \tag{35}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
i=\frac{v_{i c m}}{1 / g_{m}+2 R_{S S}} \tag{36}
\end{equation*}
$$

The voltages at the drain of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ can now be found as

$$
\begin{equation*}
v_{O 1}=v_{o 2}=-R_{D} i \tag{37}
\end{equation*}
$$

Resulting in

$$
\begin{equation*}
v_{o 1}=v_{o 2}=-\frac{R_{D}}{1 / g_{m}+2 R_{S S}} v_{i c m} \tag{38}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
\frac{v_{o 1}}{v_{i c m}}=\frac{v_{o 2}}{v_{i c m}} \simeq-\frac{R_{D}}{2 R_{S S}} \tag{39}
\end{equation*}
$$

where we have assumed that $2 \mathbb{R}_{S S} \geqslant 1 / g_{m}$ Nevertheless, because $\mathrm{V}_{01}=\mathrm{V}_{02}$ the differential output voltage $\mathrm{V}_{\text {od }}$ will remain free of common-mode interference

$$
\begin{equation*}
v_{o d}=v_{o 2}-v_{o 1}=0 \tag{40}
\end{equation*}
$$

When the two drain resistances exhibit a mismatch $\Delta R_{D}$ as they inevitably do, the common-mode voltages at the two drains will no longer be equal. Rather, if the load of $\mathrm{Q}_{1}$ is $\mathrm{R}_{\mathrm{D}}$ and that of $\mathrm{Q}_{2}$ is $\left(R_{D}+\Delta R_{D}\right)$ is the drain signal voltages arising from $\mathrm{V}_{\text {icm }}$ will be

$$
\begin{equation*}
v_{o 1} \simeq-\frac{R_{D}}{2 R_{S S}} v_{i \mathrm{~cm}} \tag{41}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{o 2} \simeq-\frac{R_{D}+\Delta R_{D}}{2 R_{S S}} v_{i c m} . \tag{42}
\end{equation*}
$$

Thus

$$
\begin{equation*}
v_{o d}=v_{o 2}-v_{o 1}=-\frac{\Delta R_{D}}{2 R_{S S}} v_{i c m} \tag{43}
\end{equation*}
$$

and we can find the common-mode gain $\mathrm{A}_{\mathrm{CM}}$ as

$$
\begin{equation*}
A_{c m} \equiv \frac{v_{o d}}{v_{i c m}}=-\frac{\Delta R_{D}}{2 R_{S S}} \tag{44}
\end{equation*}
$$

which can be expressed in the alternate form

$$
\begin{equation*}
A_{c m}=-\left(\frac{R_{D}}{2 R_{S S}}\right)\left(\frac{\Delta R_{D}}{R_{D}}\right) . \tag{45}
\end{equation*}
$$

- It follows that a mismatch in the drain resistances causes the differential amplifier to have a finite common-mode gain.
- Thus, a portion of the interference or noise signal $\mathrm{V}_{\mathrm{icm}}$ will appear as a component of $\mathrm{V}_{\text {od }}$.
- A measure of the effectiveness of the differential amplifier in amplifying differential-mode signals and rejecting common-mode interference is the ratio of the magnitude of its differential gain $\left(\mathrm{A}_{\mathrm{d}}\right)$ to the magnitude of its commonmode gain $\left(\mathrm{A}_{\mathrm{CM}}\right)$ This ratio is termed common-mode rejection ratio (CMRR).

Thus,

$$
\begin{equation*}
\mathrm{CMRR} \equiv \frac{\left|A_{d}\right|}{\left|A_{c m}\right|} \tag{46}
\end{equation*}
$$

and is usually expressed in decibels

$$
\begin{array}{r}
\mathrm{CMRR}(\mathrm{~dB})=20 \log \frac{\left|A_{d}\right|}{\left|A_{c m}\right|} \\
\mathrm{CMRR}=\left(2 g_{m} R_{S S}\right) /\left(\Delta R_{D} / R_{D}\right) \mid \ldots \tag{48}
\end{array}
$$

## Effect of $\boldsymbol{g}_{\mathrm{m}}$ mismatch on CMRR

Another possible mismatch between the two halves of the MOS differential pair is a mismatch in $\mathrm{g}_{\mathrm{m}}$ of the two transistors. For the purpose of finding the effect of a $g_{m}$ mismatch on CMRR, let

$$
\begin{align*}
g_{m 1} & =g_{m}+\frac{1}{2} \Delta g_{m}  \tag{49}\\
g_{m 2} & =g_{m}-\frac{1}{2} \Delta g_{m} \ldots \tag{50}
\end{align*}
$$

That is

$$
\begin{equation*}
g_{m 1}-g_{m 2}=\Delta g_{m} \tag{51}
\end{equation*}
$$



Fig.3.9: Analysis of the MOS differential amplifier with an input common-mode signal $v_{\mathrm{icm}}$ in the case the two transistors have a $g_{\mathrm{m}}$ mismatch.

Since the circuit is no longer symmetrical, we cannot employ the commonmode half circuit. Rather, we shall return to the original circuit of Fig.3.8 (a) and replace each of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ and with its T equivalent-circuit model. The result is the equivalent circuit shown in Fig.3.9.

Thus,

$$
\begin{align*}
& i_{1}\left(1 / g_{m 1}\right)=i_{2}\left(1 / g_{m 2}\right)  \tag{52}\\
& i_{1}+i_{2}=i_{1}\left(1+\frac{g_{m 2}}{g_{m 1}}\right) \ldots \tag{53}
\end{align*}
$$

### 3.3 Frequency response of resistively loaded MOS differential amplifier

- The basic resistively loaded MOS differential pair shown in Fig. 3.10(a).It is shown the transistor Qs that supplies the bias current I. Although we are showing a dc bias voltage $V_{\text {Bias }}$ at its gate, usually Qs is part of a current mirror.
- The total impedance between node S and ground, $\mathrm{Z}_{\mathrm{SS}}$ plays a significant role in determining the common-mode gain and the CMRR of the differential amplifier.
- Resistance $\mathrm{R}_{\mathrm{ss}}$ is simply the output resistance of current source $\mathrm{Q}_{\mathrm{s}}$. Capacitance $\mathrm{C}_{\mathrm{ss}}$ is the total capacitance between node S and ground and includes $\mathrm{C}_{\mathrm{db}}$ and $\mathrm{C}_{\mathrm{gd}}$ of $\mathrm{Q}_{\mathrm{s}}$, as well as $\mathrm{C}_{\text {sbl }}$, and $\mathrm{C}_{\text {sb2 }}$. This capacitance can be significant, especially if wide transistors are used for $\mathrm{Qs}_{\mathrm{s}}, \mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$.
- The differential half-circuit shown in Fig. 3.10 (b) can be used to determine the frequency dependence of the differential gain $V_{0} / V_{i d}$. Indeed the gain function $A_{d}(s)$ of the differential amplifier will be identical to the transfer function of this commonsource amplifier.


Fig.3.10(a) A resistively loaded MOS differential pair with the transistor supplying the bias currentexplicitly shown. It is assumed that the total impedance between node 5 and ground, $Z_{S S}$, consists of a resistance $R_{S s i n}$ parallel with a capacitance $C_{S S}$. (b) Differential half-circuit, (c) Common-mode half-circuit.

- The common-mode half-circuit is shown in Fig.3.10 (c). Although this circuit has other capacitances, namely $\mathrm{C}_{\mathrm{gs}}, \mathrm{C}_{\mathrm{gd}}$, and $\mathrm{C}_{\mathrm{db}}$ of the transistor in addition to other stray capacitances, we have chosen to show only $\mathrm{C}_{\mathrm{ss}} / 2$. This is because ( $\mathrm{C}_{\mathrm{ss}} / 2$ ) together with ( $2 \mathrm{R}_{\mathrm{ss}}$ ) forms a real-axis zero in the common-mode gain function at a frequency much lower than those of the other poles and zeros of the circuit. This zero then dominates the frequency dependence of $\mathrm{A}_{\mathrm{cm}}$ and CMRR.
- If the output of the differential amplifier is taken single-endedly, then the commonmode gain of interest is $\mathrm{V}_{\mathrm{ocm}} / \mathrm{V}_{\mathrm{icm}}$. More typically, the output is taken differentially.
- Consider what happens when the output is taken differentially andthere is a mismatch $\Delta \mathrm{R}_{\mathrm{D}}$ between the two drain resistances.

$$
\begin{equation*}
A_{C n t}=-\left(\frac{R_{D}}{2 R_{S S}}\right) \frac{\Delta R_{D}}{R_{D}} \tag{54}
\end{equation*}
$$

which is simply the product of $V_{\text {ocm }} / V_{\text {icmand }}$ the per-unit mismatch $\left(\Delta \mathrm{R}_{\mathrm{D}} / R_{D}\right)$.

- Thus, the frequency dependence of $A_{\text {cm }}$ can be obtained by simply replacing $R_{\mathrm{SS}}$ by $Z_{\text {Ssin }}$ this factor. Doing sofor the expression in Eq. (54) gives

$$
\begin{align*}
A_{c m}(s) & =-\frac{R_{D}}{2 Z_{s s}}\left(\frac{\Delta R_{D}}{R_{D}}\right) \\
& =-\frac{1}{2} R_{D}\left(\frac{\Delta R_{D}}{R_{D}}\right) Y_{s s} \\
& =-\frac{1}{2} R_{D}\left(\frac{\Delta R_{D}}{R_{D}}\right)\left(\frac{1}{R_{S S}}+s C_{S S}\right) . \\
& =-\frac{R_{D}}{2 R_{S S}}\left(\frac{\Delta R_{D}}{R_{D}}\right)\left(1+s C_{S s} R_{s s}\right) \tag{55}
\end{align*}
$$

from which we see that $\mathrm{A}_{\mathrm{cm}}$ acquires a zero on the negative real-axis of the s-plane withfrequency $w_{z}$,

$$
\begin{equation*}
\omega_{z}=\frac{1}{C_{s s} R_{s s}} \tag{56}
\end{equation*}
$$

$$
\begin{equation*}
f_{Z}=\frac{\omega_{Z}}{2 \pi}=\frac{1}{2 \pi C_{S S} R_{S S}} \tag{57}
\end{equation*}
$$

- Usually $f_{z}$ is much lower than the frequencies of the other poles and zeros. As a result, the common-mode gain increases at the rate of +6 dB /octave ( 20 dB /decade) starting at a relatively low frequency, as indicated in Fig. 3.11(a).
- $A_{\text {cm }}$ drops off at high frequencies because of the other poles of the common-mode half-circuit. It is, however, $f_{z}$ that is significant, for it is the frequency at which the CMRR of the differential amplifier begins to decrease, as indicated in Fig. 3.11(c).


Fig.3.11: Variation of (a) common-mode gain, (b) differential gain, and (c) commonmode rejectionratio with frequency.

- If both $A_{d}$ and $A_{\text {cm }}$ are expressed and plotted in dB , then CMRR in dB is simply the difference between $A_{d}$ and $A_{c m}$.
- In order to operate this current source with a small $\mathrm{V}_{\mathrm{DS}}$ (to conserve the already low $\left.V_{\mathrm{DD}}\right)$, we desire to operate the transistor at a low overdrive voltage Vov.
- For a given value of the current $I$, however, this means using a large $\mathrm{W} / \mathrm{L}$ ratio (i.e., a wide transistor). This in turn increases $C_{s S}$ and hence lowers $f_{Z}$ with the result that the CMRR deteriorates- (i.e., decreases) at a relatively low frequency.
- Thus there is atrade-off between the need to reduce the dc voltage across $Q_{\text {sand }}$ the need to keep theCMRR reasonably high at higher frequencies.
- To appreciate the need for high CMRR at higher frequencies, consider the situationillustrated in Fig.3.12


Fig.3.12: The second stage in a differential amplifier is relied on to suppress highfrequency noiseinjected by the power supply of the first stage, and therefore must maintain a high CMRR at higher frequencies.

- We show two stages of a differential amplifier whose power-supplyvoltage $V_{D D}$ is corrupted with high-frequency noise.
- Since the quiescent voltage at each of the drains of $Q_{1}$ and $Q_{2}$ is $\left[V_{D D}-(I / 2) R_{D}\right]$, we see that $v_{D 1}$ and $v_{D 2}$ will have the same high frequencynoise as $V_{D D}$.
- This high-frequency noise then constitutes a common-mode input signal to the second differential stage, formed by $Q_{3}$ and $Q_{4}$.
- If the second differential stage is perfectly matched, its differential output voltage $V_{o s h o u l d ~ b e ~ f r e e ~ o f ~ h i g h-f r e q u e n c y n o i s e . ~}^{\text {se }}$
- In practice there is no such thing as perfect matching and the second stage will have a finite common-mode gain. Because of the zero formed by $R_{S S}$ and $C_{S S}$ of the second stage, the common-mode gain will increase with frequency, causing some of the noise to make its way to $V$. With careful design, this undesirable component of $V_{o}$ can be kept small.


### 3.4 BJT Differential pair

The basic BJT differential-pair configuration is shown in figure 3.13. It is very similar to theMOSFET circuit and consists of two matched transistors, $Q_{1}$ and $Q_{2}$, whose emitters arejoined together and biased by a constant-current source $I$.


Fig.3.13: The basic BJT differential pair configuration.

### 3.4.1 Basic Operation

The BJTdifferential-pair also works in two modes Common- mode and differential mode. Consider the case acommon-mode input voltage $V_{C M}$.


Fig.3.14: (a) The differential pair with a common-mode input voltage $V_{C M}$

- In this the two bases joined together and connected to a common-mode voltage $V_{C M}$, i.e $v_{B 1}=v_{B 2}=V_{C M}$.
- Since $Q_{1}$ and $Q_{2}$ are matched, and assuming an ideal bias current source $I$ with infinite output resistance, it follows that the current $I$ will remain constant and from symmetry that $I$ will divide equally between the two devices.
- Thus $i_{E 1}=i_{E 2}=\mathrm{I} / 2$, and the voltage at the emitters will be $V_{C M}-V_{B E}$, where $V_{B E}$ is the base-emitter voltage corresponding to an emitter current of I/2. The voltage at each
collector will be $\mathrm{V}_{\mathrm{CC}}-(1 / 2)$ a I $\mathrm{R}_{\mathrm{C}}$, and the difference in voltage between the two collectors will be zero.
- Now let us vary the value of the common-mode input voltage $V_{C M}$. Obviously, as long as $Q_{1}$ and $Q_{2}$ remain in the active region and the current source $I$ has sufficient voltage acrossit to operate properly, the current $I$ will still divide equally between $Q_{1}$ and $Q_{2}$, and the voltages at the collectors will not change.
- Thus the differential pair does not respond to (i.e., it rejects) changes in the common-mode input voltage.


### 3.4.2 The differential pair with a "large" differential input signal

## Case (i)

Let the voltage $v_{B 2}$ be set to a constant value, say, zero (by grounding $B_{2}$ ), and let $v_{B 1}=+1 \mathrm{~V}=>\mathrm{V}_{\mathrm{id}}>0$

- It can be seen that $Q_{1}$ will be on and conducting all of the current $I$ and that $Q_{2}$ will be off.
- For $Q_{1}$ to be on (with $V_{B E 1}=0.7 \mathrm{~V}$ ), the emitter has to be at approximately +0.3 V, which keeps the EBJ of $Q_{2}$ reverse-biased.
- The collector voltages will be $v_{C 1}=V_{C C}-$ a $I R C$ and $v_{C 2}=V_{C C}$.


Fig.3.14: (b) the differential pair with a "large" differential input signal; with $\mathrm{v}_{\mathrm{B} 1}=+1 \mathrm{~V}$ and $\mathrm{v}_{\mathrm{B} 2}=0 \mathrm{~V}$

- In this case $\mathrm{Q}_{1}$ conducts entire current, $\mathrm{Q}_{2}$ is off. Therefore, The collector voltages will be $v_{C 1}=V_{C C}-$ a $I R_{C}$ and $v_{C 2}=V_{C C}$.


## Case (ii)

Let the voltage $v_{B 2}$ be set to a constant value, say, zero (by grounding $B_{2}$ ), and let $v_{B 1}=-1 \mathrm{~V}=>\mathrm{v}_{\mathrm{id}}<0$.


Fig.3.14: (c) the differential pair with a "large" differential input signal; with $\mathrm{v}_{\mathrm{B} 1}=-1 \mathrm{~V}$ and $\mathrm{v}_{\mathrm{B} 2}=0 \mathrm{~V}$

- Again with some reasoning it can be seen that $Q_{1}$ will turn off, and $Q_{2}$ will carry all the current $I$.
- The common emitter will be at -0.7 V , which means that the EBJ of $Q_{1}$ will be reverse biased by 0.3 V . The collector voltages will be $v_{C 1}=V_{C C}$ and $v_{C 2}=V_{C C}-\mathrm{a}$ $I R_{C}$.


### 3.4.3 The differential pair with a "small" differential input signal



Fig.3.15: The differential pair with a "small" differential input signal applied

- To use the BJT differential pair as a linear amplifier, a very small differential signal (a few millivolts), which will result in one of the transistors conducting a current $\frac{I}{2}+\Delta I$
- The current in the other transistor will be $\frac{I}{2}-\Delta I$, with $\Delta I$ being proportional to the difference input voltage. The output voltage taken between the two collectors will be $2 \mathrm{a} \Delta I R_{C}$, which is proportional to the differential input signal $v_{i}$.


## Input Common-Mode Range

The allowable range of is determined at the upper end by and leaving the active mode and entering saturation.
Thus

$$
V_{C M \max } \approx V_{C}+0.4=V_{C C}-\alpha \frac{I}{2} R_{C}+0.4
$$

The lower end of the range is determined by the need to provide a certain minimumvoltage across the current source $I$ to ensure its proper operation. Thus,

$$
V_{C M \min }=-V_{E E}+V_{C S}+V_{B E}
$$

## Large-Signal Operation

Assuming $Q_{1} \& \mathrm{Q}_{2}$ matched transistors with Bias Emitter current of I/2.


As we know,

$$
\begin{aligned}
& i_{E 1}=\frac{I_{S}}{\alpha} e^{\left(v_{B 1}-v_{B}\right) / V_{T}} \\
& i_{E 2}=\frac{I_{S}}{\alpha} e^{\left(v_{B 2}-v_{B}\right) / V_{T}}
\end{aligned}
$$

These two equations can be combined to obtain

$$
\frac{i_{E 1}}{i_{E 2}}=e^{\left(v_{B 1}-v_{B 2}\right) / V_{T}}
$$

which can be manipulated to yield
which can be manipulated to yield

$$
\begin{aligned}
& \frac{i_{E 1}}{i_{E 1}+i_{E 2}}=\frac{1}{1+e^{\left(v_{B 2}-v_{B 1}\right) / V_{T}}} \\
& \frac{i_{E 2}}{i_{E 1}+i_{E 2}}=\frac{1}{1+e^{\left(v_{B 1}-v_{B 2}\right) / V_{T}}}
\end{aligned}
$$

The circuit imposes the additional constraint

$$
i_{E 1}+i_{E 2}=I
$$

Substituting $v_{B 1}-v_{B 2}=v_{i d}$ gives

$$
\begin{aligned}
& i_{E 1}=\frac{I}{1+e^{-v_{i d} / V_{T}}} \\
& i_{E 2}=\frac{I}{1+e^{v_{i d} / V_{T}}}
\end{aligned}
$$



Normalized differential input voltage, $\frac{v_{\text {id }}}{V_{T}}$

Fig.3.16: Transfer characteristics of the BJT differential pair, assuming $\alpha \cong-1$

## Small-Signal Operation

- The BJT differential pair with a difference voltage signal $\mathrm{v}_{\mathrm{id}}$ applied between the two bases.
- For instance, one of the two input terminals can be grounded and $\mathrm{v}_{\mathrm{id}}$ applied to the other input terminal. Alternatively, the differential amplifier may be fed from the output of another differential amplifier.
- In thelatter case, the voltage at one of the input terminals will be $V C M+\frac{v i d}{2}$ while that at the other input terminal will be VCM $-\frac{v i d}{2}$.


Fig.3.17:The currents and voltages in the differential amplifier when a small differential input signalvidis applied.

## The collector currents when vidisapplied

$$
\begin{aligned}
i_{C 1} & =\frac{\alpha I}{1+e^{-v_{i d} / V_{T}}} \\
i_{C 2} & =\frac{\alpha I}{1+e^{v_{i d} / V_{T}}}
\end{aligned}
$$

Multiplying the numerator and the denominator of the right-hand side

$$
i_{C 1}=\frac{\alpha I e^{v_{i d} / 2 V_{T}}}{e^{v_{i d} / 2 V_{T}}+e^{-v_{i d} / 2 V_{T}}}
$$

Assume that $v i_{d} \ll 2 V_{T}$. We may thus expand the exponential in a series and retainonly the first two terms:

$$
i_{C 1} \simeq \frac{\alpha I\left(1+v_{i d} / 2 V_{T}\right)}{1+v_{i d} / 2 V_{T}+1-v_{i d} / 2 V_{T}}
$$

Thus

$$
\begin{align*}
& i_{C 1}=\frac{\alpha I}{2}+\frac{\alpha I}{2 V_{T}} \frac{v_{i d}}{2}  \tag{1}\\
& i_{C 2}=\frac{\alpha \grave{\dagger}}{2}-\frac{\alpha I}{2 V_{T}} \frac{v_{i d}}{2} \tag{2}
\end{align*}
$$

- Equations (1) \& (2) tell us that when vid $=0$, the bias current I divides equally between the two transistors of the pair.
- Thus each transistor is biased at an emitter current of I/2. When a "smallsignal" vid is applied differentially (i.e., between the two bases), the collector current. of $Q_{1}$ increases by an increment $i_{c}$ and that of $Q_{2}$ decreases by an equal amount.
- This ensures that the sum of the total currents in $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ remains constant, as constrained by the currentsource bias. The incremental (or signal) current component $i_{c}$ is given by

$$
i_{c}=\frac{\alpha I}{2 V_{T}} \frac{v_{i d}}{2}
$$

Thus the total base-emitter voltages will be

$$
\begin{aligned}
& \left.v_{B E}\right|_{Q 1}=V_{B E}+\frac{v_{i d}}{2} \\
& \left.v_{B E}\right|_{Q 2}=V_{B E}-\frac{v_{i d}}{2}
\end{aligned}
$$

Where $V_{B E i s}$ the dc BE voltage corresponding to an emitter current of I/2. Therefore, the collector current of $Q_{1}$ will increase by $g m \frac{v i d}{2}$ and the collector current of $Q_{2}$ will decrease by $\frac{v i d}{2}$. Here $g_{m}$ denotes the transconductance of $Q_{1}$ and of $Q_{2}$, which are equal and given by

$$
g_{m}=\frac{I_{C}}{V_{T}}=\frac{\alpha I / 2}{V_{T}}
$$

## Differential Voltage Gain

$$
\begin{aligned}
& i_{C 1}=I_{C}+g_{m} \frac{v_{i d}}{2} \\
& i_{C 2}=I_{C}-g_{m} \frac{v_{i d}}{2}
\end{aligned}
$$

Where

$$
I_{C}=\frac{\alpha I}{2}
$$

Thus the total voltages at the collectors will be

$$
\begin{aligned}
& v_{C 1}=\left(V_{C C}-I_{C} R_{C}\right)-g_{m} R_{C} \frac{v_{i d}}{2} \\
& v_{C 2}=\left(V_{C C}-I_{C} R_{C}\right)+g_{m} R_{C} \frac{v_{i d}}{2}
\end{aligned}
$$

- The quantities in parentheses are simply the dc voltages at each of the two collectors.
- As in the MOS case, the output voltage signal of a bipolar differential amplifier can be taken differentially (i.e., between the two collectors, $\mathrm{v}_{\mathrm{od}}=\mathrm{v}_{\mathrm{C} 2}-\mathrm{v}_{\mathrm{C} 1}$ ).
- The differential gain of the differential amplifier will be

$$
A_{d}=\frac{v_{o d}}{v_{i d}}=g_{m} R_{C}
$$

For the differential amplifier with resistances in the emitter leads,

$$
A_{d}=\frac{\alpha\left(2 R_{C}\right)}{2 r_{e}+2 R_{e}} \simeq \frac{R_{C}}{r_{e}+R_{e}}
$$

## Common-Mode Gain and CMRR

A bipolar differential amplifier with an input common-mode signal is shown in figure 3.18 (a).

- Here is the output resistance of the bias current source $I$. We wish to find thevoltages that result from at the collectors of $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}, v_{O_{1}}$ and $v_{02}$ between the two collectors.
- Toward that end, we make use of the common-mode half-circuits.
- The signal $v_{01}$ that appears at the collector of in response to will be

$$
v_{o 1}=-\frac{\alpha R_{C}}{r_{e}+2 R_{E E}} v_{i c m}
$$

Similarly vo2, will be

$$
v_{o 2}=-\frac{\alpha R_{C}}{r_{e}+2 R_{E E}} v_{i c m}
$$

where we have neglected the transistor for simplicity. The differential output signalcan be obtained as

$$
v_{o d}=v_{o 2}-v_{o 1}=0
$$


(a)

(b)

Fig.3.18: (a) The differential amplifier fed by a common-mode input signal $v_{i c m}$. (b)Equivalent"half-circuits" for common-mode calculations.

- Thus, while the voltages at the two collectors will contain common-mode noise or interference components, the output differential voltage will be free from such interference.
- This condition, however, is based on the assumption of perfect matching between the two sides of the differential amplifier. Any mismatch will result inv ${ }_{\text {od }}$ acquiring a component proportionaltov ${ }_{i c m}$.


## Considering $\mathbf{R}_{\mathrm{C}}$ Mismatch

Assuming there is a mismatch $\Delta R c$ between the two collectorresistances: If the collector of $\mathrm{Q}_{1}$ has a collector resistance $\mathrm{R}_{\mathrm{C}}$,

$$
v_{o 1}=-\frac{\alpha R_{C}}{2 R_{E E}+r_{e}} v_{i c m}
$$

and the collector of $Q_{2}$ has a collector resistance $\left(R_{C}+\Delta R_{C}\right)$,

$$
v_{o 2}=-\frac{\alpha\left(R_{C}+\Delta R_{C}\right)}{2 R_{E E}+r_{e}} v_{i c m}
$$

then the differential output voltage $v_{o d}$ will be

$$
\begin{aligned}
v_{o d} & \equiv v_{o 2}-v_{o 1} \\
& =-\frac{\alpha \Delta R_{C}}{2 R_{E E}+r_{e}} v_{i c m}
\end{aligned}
$$

and the common-mode gain will be

$$
\begin{aligned}
A_{c m} \equiv \frac{v_{o d}}{v_{i c m}} & =-\frac{\alpha \Delta R_{C}}{2 R_{E E}+r_{e}} \\
A_{c m} & \simeq-\left(\frac{R_{C}}{2 R_{E E}}\right)\left(\frac{\Delta R_{C}}{R_{C}}\right)
\end{aligned}
$$

The common-mode rejection ratio can now be found from

$$
\begin{gathered}
\mathrm{CMRR}=\frac{\left|A_{d}\right|}{\left|A_{c m}\right|} \\
\mathrm{CMRR}=\left(2 g_{m} R_{E E}\right) /\left(\frac{\Delta R_{C}}{R_{C}}\right)
\end{gathered}
$$

## Input Differential Resistance

- Unlike the MOS differential amplifier, which has an infiniteinput resistance, the bipolar differential pair exhibits a finite input resistance, a result of thefinite $\beta$ of the BJT.
- The input differential resistance is the resistance seen between the two bases; that is, it is the resistance seen by the differential input signal vid. For the differential amplifier inbelow fig. 3.19 (a) $\&(\mathrm{~b})$, it can be seen that the base
current of $Q_{1}$ shows an increment $i_{b}$ and the base current of $Q_{2}$ shows an equal decrement.


Fig.3.19: (a)A simple technique for determining the signal currents in a differential amplifier excited by adifferential voltage signal $v_{i d}$; dc quantities are not shown.


Fig.3.19: (b)A differential amplifier with emitter resistances.

- Thus the collector of $Q_{1}$ will exhibit a current increment $i_{c}$ and the collector of $Q_{2}$ will exhibit a current decrement $i_{c}$ :

$$
i_{c}=\alpha i_{e}=\frac{\alpha v_{i d}}{2 r_{e}}=g_{m} \frac{v_{i d}}{2}
$$

Each transistor is biased at an emitter current of I/2.

$$
i_{e}=\frac{v_{i d}}{2 r_{e}+2 R_{e}}
$$

$$
i_{b}=\frac{i_{e}}{\beta+1}=\frac{v_{i d} / 2 r_{e}}{\beta+1}
$$

Thus the differential input resistance $R i_{d}$ is given by

$$
R_{i d} \equiv \frac{v_{i d}}{i_{\mathrm{k}}}=(\beta+1) 2 r_{e}=2 r_{\pi}
$$

This result is just a restatement of the familiar resistance-reflection rule; namely, the resistanceseen between the two bases is equal to the total resistance in the emitter circuit multiplied by $(\beta+1)$.

$$
R_{i d}=(\beta+1)\left(2 r_{e}+2 R_{e}\right)
$$

## INTRODUCTION TO OP-AMPS AND TUNED AMPLIFIERS

## Objectives:

$>$ To introduce the students about basic block diagram, symbol, specifications and characteristics of an operational amplifier.
$>$ To familiarize with circuit configuration and analysis of tuned amplifiers

SYLLABUS: Ideal op amp, inverting and non-inverting configurations, Tuned amplifiers.
Pre-requisites: Electronic Devices, differential amplifiers

## Outcomes:

Students will be able to
$>$ Identify the symbol and equivalent notation of an operational amplifier.
$>$ Distinguish between ideal and practical specifications of an operational amplifier.
$>$ Design circuits using operational amplifiers for various linear applications.
$>$ To draw and analyze the behaviour of tuned amplifiers and op amp under inverting and noninverting configurations.

### 4.1 THE IDEAL OP AMP

4.1.1 The Op-Amp Terminals

- From a signal point-of-view the op amp has three terminals: two input terminals and one output terminal.
- Below Figure shows the symbol we shall use to represent the op amp. Terminals 1 and 2 are input terminals, and terminal 3 is the output terminal.


Circuit symbol for the op amp

- Below Figure shows two terminals, 4 and 5, are brought out of the op-amp package and connected to a positive voltage Vcc and a negative voltage -VEE, respectively. In Fig. (b).


The op amp shown connected to de power sopplies.

- we explicitly show the two dc power supplies as batteries with a common ground. It is interesting to note that the reference grounding point in op-amp circuits is just the common terminal of the two power supplies; that is, no terminal of the op-amp package is physically connected to ground.
- In addition to the three signal terminals and the two power-supply terminals, an op amp may have other terminals for specific purposes. These other terminals can include terminals for frequency compensation and terminals for offset nulling;


### 4.1.2 Function and Characteristics of the Ideal Op Amp

- We now consider the circuit function of the op amp. The op amp is designed to sense the difference between the voltage signals applied at its two input terminals (i.e., the quantity (v2-v1), multiply this by a number A, and cause the resulting voltage $\mathrm{A}(\mathrm{v} 2-\mathrm{v} 1)$ appear at output terminal 3.
- Here it should be emphasized that when we talk about the voltage at a terminal we mean the voltage between that terminal and ground; thus v1 means the voltage applied between terminal 1 and ground.
- The ideal op amp is not supposed to draw any input current; that is, the signal current into terminal 1 and the signal current into terminal 2 are both zero. In other words, the input impedance of an ideal op amp is supposed to be infinite, the output impedance of an ideal op amp is supposed to be zero.
- Putting together all of the above, we arrive at the equivalent circuit model shown in below
- Fig. Note that the output is in phase with (has the same sign as) v2 and is out of phase with (has the opposite sign of v1. For this reason, input terminal 1 is called the inverting input terminal and is distinguished by a " - " sign, while input terminal 2 is called the non inverting input terminal and is distinguished by a "+" sign.
- As can be seen from the above description, the op amp responds only to the difference signal $\mathrm{v} 2-\mathrm{v} 1$ and hence ignores any signal common to both inputs. That is, if $\mathrm{v} 1=\mathrm{v} 2=1 \mathrm{~V}$, then the output will-ideally-be zero. We call this property common-mode rejection, and we conclude that an ideal op amp has zero common-mode gain or, equivalently, infinite common-mode rejection.
- For the time being note that the op amp is a differential-input single-endedoutput amplifier, with the latter term referring to the fact that the output appears between terminal 3 and ground. Furthermore, gain A is called the differential gain.

- An important characteristic of op amps is that they are direct-coupled or dc amplifiers, where dc stands for direct-coupled. The fact that op amps are directcoupled devices will allow us to use them in many important applications.
- How about bandwidth? The ideal op amp has a gain A that remains constant down to Zero frequency and up to infinite frequency. That is, ideal op amps will amplify signals of any frequency with equal gain, and are thus said to have infinite bandwidth.
- We have discussed all of the properties of the ideal op amp except for one, which in fact is the most important. This has to do with the value of A. The ideal op amp should have a gain A whose value is very large and ideally infinite.


### 4.1.3 Differential and Common-Mode Signals

The differential input signal $v_{I d}$ is simply the difference between the two input signals $v_{l}$ and $v_{2}$; that is, $\boldsymbol{V}_{\text {id }}=\boldsymbol{V}_{\mathbf{2}}-\boldsymbol{V}_{\mathbf{1}}$
The common-mode input signal $v_{\text {Icm }}$ is the average of the two input signals $v l$ and $v 2$ namely, $v_{\text {Icm }}=1 / 2(v 1+v 2)$
Above equations can b e used to express the input signals $v_{1}$ and $v_{2}$ in terms of their differential and common-mode components as follows:

$$
v_{l}=v \mathrm{lcm}-v i d / 2
$$

and

$$
v_{2}=v I c m+v \text { id } / 2
$$

These equations can in turn lead to the pictorial representation in below Fig.


Representation of the signal source v 1 and v 2 in terms of their differential and common mode components

### 4.2 THE INVERTING CONFIGURATION

As mentioned above, op amps are not used alone; rather, the op amp is connected to passive components in a feedback circuit. There are two such basic circuit configurations employing an op amp and two resistors: the inverting configuration, which is studied in this section, and the no inverting configuration,
Below Figure shows the inverting configuration. It consists of one op amp and two resistors R1 and $R 2$. Resistor $R 2$ is connected from the output terminal of the op amp, terminal 3, back to the inverting or negative input terminal, terminal 1 . We speak of $R 2$ as applying negative feedback; if $R 2$ were connected between terminals 3 and 2 we would have called this positive feedback.

Note also that $R 2$ closes the loop around the op amp. In addition to adding $R 2$, we have grounded terminal 2 and connected a resistor $R 1$ between terminal 1 and an input signal source


The inverting closed loop configuration
with a voltage $v_{l}$. The output of the overall circuit is taken at terminal 3 (i.e., between terminal 3 and ground). Terminal 3 is, of course, a convenient point to take the output, since the impedance level there is ideally zero. Thus the voltage $v 0$ will not depend on the value of the current that might be supplied to a load impedance connected between terminal 3 and ground.

### 4.2.1 The Closed-Loop Gain

The closed-loop gain $\mathbf{G}$, defined as $\mathrm{G}=\mathrm{Vo} / \mathrm{Vi}$ We will do so assuming the op amp to be ideal. Below figure (a) shows the equivalent circuit and the analysis proceeds as follows: The gain A is very large (ideally infinite). If we assume that the circuit is "working" and producing a finite output voltage at
terminal 3, then the voltage between the op amp input terminals should be negligibly small and ideally zero.
Specifically, if we call the output voltage $v 0$, then, by definition,

$$
\mathrm{V} 2-\mathrm{V} 1=\mathrm{Vo} / \mathrm{A}=0
$$

It follows that the voltage at the inverting input terminal $\left(v_{1}\right)$ is given by $v_{l}=v_{2}$. That is, because the gain A approaches infinity, the voltage $v_{1}$ approaches and ideally equals $v_{2}$. We speak of this as the two input tenninals "tracking each other in potential." We also speak of a "virtual short circuit" that exists between the two input terminals. Here the word virtual should be emphasized, and one should not make the mistake of physically shorting terminals 1 and 2 together while analyzing a circuit. A virtual short circuit means that whatever voltage is at 2 will automatically appear at 1 because of the infinite gain A. But terminal 2 happens to be connected to ground; thus $v_{2}=0$ and $v_{l}=0$. We speak of terminal 1 as being a virtual ground-
that is, having zero voltage but not physically connected to ground Now that we have determined $v_{l}$ we are in a position to apply Ohm's law and find the current $i_{1}$ through $R_{l}$

$$
\begin{gathered}
i_{1}=\frac{v_{I}-v_{1}}{R_{1}}=\frac{v_{I}-0}{R_{1}}=\frac{v_{I}}{R_{1}} \\
v_{O}=v_{1}-i_{1} R_{2} \\
=0-\frac{v_{1}}{R_{1}} R_{2} \quad \frac{v_{O}}{v_{I}}=-\frac{R_{2}}{R_{1}}
\end{gathered}
$$



Fig: Analysis of the inverting configuration
Below Figure (b) illustrates these steps and indicates by the circled numbers the order in which the analysis is performed.
We thus see that the closed-loop gain is simply the ratio of the two resistances $R 2$ and $R 1$. The minus sign means that the closed-loop amplifier provides signal inversion.

The fact that the closed-loop gain depends entirely on external passive components (resistors Rl and $R 2$ ) is very significant. It means that we can make the closed-loop gain as accurate as we want by selecting passive components of appropriate accuracy. It also means that the closed-loop gain is (ideally) independent of the op-amp gain. This is a dramatic illustration of negative feedback: We started out with an amplifier having very large gain A, and through applying negative feedback we have obtained a closed-loop gain $R 2 / R 1$ that is much smaller than $A$ but is stable and predictable. That is, we are trading gain for accuracy.

### 4.2.2 Effect of Finite Open-Loop Gain

The points just made are more clearly illustrated by deriving an expression for the closed loop gain under the assumption that the op-amp open-loop gain A is finite. Below figure shows the analysis. If we denote the output voltage $v_{0}$, then the voltage between the two input terminals of the op amp will be $v_{0} / A$. Since the positive input terminal is grounded, the voltage at the negative input terminal must be $-v_{0} / A$. The current $i x$ through $R x$ can now be found from

$$
i_{1}=\frac{v_{I}-\left(-v_{0} / A\right)}{R_{1}}=\frac{v_{I}+v_{0} / A}{R_{1}}
$$

The infinite input impedance of the op amp forces the current $i_{1}$ to flow entirely through $R 2$. The output voltage $v_{0}$ can thus be determined from

$$
\begin{aligned}
v_{O} & =-\frac{v_{O}}{A}-i_{1} R_{2} \\
& =-\frac{v_{O}}{\Lambda}-\left(\frac{v_{I}+v_{O} / A}{R_{1}}\right) R_{2}
\end{aligned}
$$

Collecting terms, the closed-loop gain $G$ is found as

$$
G \equiv \frac{v_{O}}{v_{I}}=\frac{-R_{2} / R_{1}}{1+\left(1+R_{2} / R_{1}\right) / A}
$$

We note that as A approaches infinity, $G$ approaches the ideal value of $-R_{2} / R_{l}$. Also, from above Fig. we see that as A approaches infinity, the voltage at the inverting input terminal approaches zero. This is the virtual-ground assumption we used in our earlier analysis when the op amp was assumed to be ideal.

$$
1+\frac{R_{2}}{R_{1}} \leqslant A
$$


fig:Analysis of the inverting configuation taking into account the infinite open loop gain of the op amp

### 4.2.3 Input and Output Resistances

Assuming an ideal op amp with infinite open-loop gain, the input resistance of the closed loop inverting amplifier is simply equal to $R x$. This can be seen

$$
R_{i} \equiv \frac{v_{l}}{i_{1}}=\frac{v_{l}}{v_{l} / R_{1}}=R_{1}
$$

we learned that the amplifier input resistance forms a voltage divider with the resistance of the source that feeds the amplifier. Thus, to avoid the loss of signal strength, voltage amplifiers are required to have high input resistance. In the case of the inverting op-amp configuration we are studying, to make $R i$ high we should select a high value for $R x$. However, if the required gain $R 2 / R 1$ is also high, then $R 2$
could become impractically large (e.g., greater than a few mega ohms). We may conclude that the inverting configuration suffers from a low input resistance.

### 4.2.4 An Important Application-The Weighted Summer

A very important application of the inverting configuration is the weighted-summer circuit shown in below Fig. Here we have a resistance $\mathrm{R}_{\mathrm{f}}$ in the negative-feedback path (as before), but we have a number of input signals $v_{1}, v_{2}, \ldots, v n$ each applied to a corresponding resistor $R_{1}, R_{2}, \ldots, R n$, which are connected to the inverting terminal of the op amp. From our previous discussion, the ideal op amp will have a virtual ground appearing at its negative input terminal. Ohm's law then tells us that the currents i, , $i 2, \ldots, i$, are given by

$$
i_{1}=\frac{v_{1}}{R_{1}}, \quad i_{2}=\frac{v_{2}}{R_{2}}, \quad \cdots, \quad i_{n}=\frac{v_{n}}{\dot{R}_{n}}
$$

### 4.3 THE NONINVERTING CONFIGURATION

The second closed-loop configuration we shall study is shown in below Fig. Here the input signal $v_{l}$ is applied directly to the positive input terminal of the op amp while one terminal of $R_{l}$ is connected to ground.


Fig: The non inverting configuration

### 4.3.1 The Closed-Loop Gain

Analysis of the non inverting circuit to determine its closed-loop gain ( $v_{0} / v_{l}$, ) is illustrated in below Fig. Notice that the order of the steps in the analysis is indicated by circled numbers.


Fig: Analysis of non inverting configuration
Assuming that the op amp is ideal with infinite gain, a virtual short circuit exists between its two input terminals. Hence the difference input signal is

$$
v_{I d}=\frac{v_{O}}{A}=0 \quad \text { for } A=\infty
$$

Thus the voltage at the inverting input terminal will be equal to that at the non inverting input terminal, which is the applied voltage vi. The current through Ri can then be determined as $v i / R i$. Because of the infinite input impedance of the op amp, this current will flow through $R 2$, as shown in Fig. Now the output voltage can be determined from

$$
\begin{array}{r}
v_{O}=v_{I}+\left(\frac{v_{I}}{R_{\mathrm{I}}}\right) R_{2} \\
\frac{v_{O}}{v_{I}}=1+\frac{R_{2}}{R_{1}}
\end{array}
$$

Further insight into the operation of the non inverting configuration can be obtained by considering the following: Since the current into the op-amp inverting input is zero, the circuit composed of Rl and $R 2$ acts in effect as a voltage divider feeding a fraction of the output voltage back to the inverting input terminal of the op amp; that is,

$$
v_{1}=v_{O}\left(\frac{R_{1}}{R_{1}+R_{2}}\right)
$$

Then the infinite op-amp gain and the resulting virtual short circuit between the two input terminals of the op-amp forces this voltage to be equal to that applied at the positive input terminal; thus

$$
v_{o}\left(\frac{R_{1}}{R_{1}+R_{2}}\right)=v_{I}
$$

### 4.3.2 Characteristics of the Non inverting Configuration

The gain of the non inverting configuration is positive-hence the name non inverting. The input impedance of this closed-loop amplifier is ideally infinite, since no current flows into the positive input terminal of the op amp. The output of the non inverting amplifier is taken at the terminals of the ideal voltage source $A\left(v_{2}-v_{1}\right.$,) thus the output resistance of the non inverting configuration is zero.

### 4.3.3 Effect of Finite Open-Loop Gain

As we have done for the inverting configuration, we now consider the effect of the finite op-amp open-loop gain A on the gain of the non inverting configuration. Assuming the op amp to be ideal except for having a finite open-loop gain A , it can be shown that the closed-loop gain of the non inverting amplifier circuit is given by

$$
G \equiv \frac{v_{\underline{O}}}{v_{1}}=\frac{1+\left(R_{2} / R_{1}\right)}{1+\frac{1+\left(R_{2} / R_{1}\right)}{A}}
$$

Observe that the denominator is identical to that for the case of the inverting configuration This is no coincidence; it is a result of the fact that both the inverting and the non inverting configurations have the same feedback loop, which can be readily seen if the input signal source is eliminated (i.e., shortcircuited). The numerators, however, are different, for the numerator gives the ideal or nominal closed-loop gain ( $-R 2 / R 1$ for the inverting configuration, and $1+R 2 / R 1$ for the non inverting
configuration). Finally, we note (with reassurance) that the gain expression reduces to the ideal value for A = infinity. In fact, it approximates the ideal value for

$$
A \gg 1+\frac{R_{2}}{R_{1}}
$$

This is the same condition as in the inverting configuration, except that here the quantity on the right-hand side is the nominal closed-loop gain.

### 4.3.4 The Voltage Follower

The property of high input impedance is a very desirable feature of the non inverting configuration. It enables using this circuit as a buffer amplifier to connect a source with a high impedance to a lowimpedance load. In many applications the buffer amplifier is not required to provide any voltage gain; rather, it is used mainly as an impedance transformer or a power amplifier. In such cases we may make $R 2=0$ and $R t={ }^{\circ}$ to obtain the unity-gain amplifier shown in Below Fig.This circuit is commonly referred to as a voltage follower, since the output "follows" the input. In the ideal case, $v 0$ $=v 1$ Rin $=\infty$, Rout $=0$, and the follower has the equivalent circuit shown in Fig. (b). Since in the voltage-follower circuit the entire output is fed back to the inverting input, the circuit is said to have $100 \%$ negative feedback. Since the non inverting configuration has a gain greater than or equal to unity, depending on the choice of $R 2 / R 1$, some prefer to call it "a follower with gain."


The unity gain buffer or follower amplifier and its equivalent circuit model

## Tuned Amplifiers:

- Tuned Amplifiers are the special kind of frequency selective networks.
- The shape of the frequency response of a tuned amplifier ( $\mathrm{f}_{0}$ ) is shown in below figure.
- Tuned Amplifiers are mainly used in video and radiofrequency applications and also in radio transmitters and receivers.
- Tuned Amplifiers behaves like narrow band pass filters with very narrow bandwidth.
- Tuned Amplifiers has a very high gain at centre frequency and gain drops at all other frequencies.



## Basic Principle:

The basic principle underlying the design of tuned amplifiers is the use of a parallel LCR circuit as the load, or at the input, of a BJT or a FET amplifier. This is illustrated in below figure, with a MOSFE T amplifier having a tuned-circuit load. For simplicity, the bias details are not included. Since this circuit uses a single tuned circuit, it is known as a single-tuned amplifier. The amplifier equivalent circuit is shown. Here R denotes the parallel equivalent of RL and the output resistance r , of the FET, and C is the parallel equivalent of CL and the FET output capacitance (usually very small). From the equivalent circuit we can write

(a)

(b)

## MOSFET based tuned amplifier and its equivalent circuit

$$
\begin{equation*}
V_{o}=\frac{-g_{\mathrm{min}} V_{i}}{Y_{L}}=\frac{-g_{i n} V_{i}}{s C+1 / R+1 / s L} \tag{1}
\end{equation*}
$$

Thus the voltage gain can be expressed as

$$
\frac{V_{o}}{V_{i}}=-\frac{g_{n n}}{C} \frac{s}{s^{2}+s(1 / C R)+1 / L C}
$$

which is a second-order band pass function. Thus the tuned amplifier has a center frequency of $\omega_{0}=1 / \sqrt{L C}$ and

Band width of
$B=\frac{1}{C R}$
And the Quality factor $(\mathrm{Q})$ is given as
$Q=\omega_{0} / B=\omega_{0} C R$

## Inductor Losses:

The power loss in the inductor is usually represented by a series resistance $r_{s}$ as shown in Fig. 3(a). However, rather than specifying the value of $r_{s}$, the usual practice is to specify the inductor Q factor at the frequency of interest,

$$
Q_{0}=\frac{\omega_{0} L}{r_{x}}
$$

The analysis of a tuned amplifier is greatly simplified by representing the inductor loss by a parallel resistance $R_{p}$, as shown. The relationship between $R_{p}$ and $Q_{0}$ can be found $b y$ writing, for the admittance of the circuit in

$$
\begin{aligned}
Y\left(j \omega_{0}\right) & =\frac{1}{r_{s}+j \omega_{0} L} \\
& =\frac{1}{j \omega_{0} L} \frac{1}{1-j\left(1 / Q_{0}\right)}=\frac{1}{j \omega_{0} L} \frac{1+j\left(1 / Q_{0}\right)}{1+\left(1 / Q_{0}^{2}\right)}
\end{aligned}
$$


(a)

(b)
(a) series R-L circuit and (b) Parallel R-L circuit

## For $\mathrm{Q}_{0} \gg 1$

$Y\left(j \omega_{0}\right)=\frac{1}{j \omega_{0} L}\left(1+j \frac{1}{\varrho_{0}}\right)$
$Q_{0}=\frac{R_{p}}{\omega_{0} L}$

## Equivalently

$R_{g}=\omega_{0} L Q_{0}$

## Use of Transformers:

In many cases it is found that the required value of inductance is not practical, in the sense that coils with the required inductance might not b e available with the required high values of $\mathrm{Q}_{0}$. A simple solution is to use a transformer to effect an impedance change. Alternatively, a tapped coil, known as an autotransformer, can be used, as shown. Provided the two parts of the inductor are tightly coupled, which can be achieved by winding on a ferrite core, the transformation relationships shown hold. The result is that the tuned circuit seen between terminals 1 and 1 ' is equivalent to .

In applications that involve coupling the output of a tuned amplifier to the input of another amplifier, the tapped coil can be used to raise the effective input resistance of the latter amplifier stage. In this way, one can avoid reduction of the overall Q. This point is illustrated in Fig. 5 and in the following exercises.


(a)

(b)
(a) The output of a tuned amplifier is coupled to the input of another amplifier via a tapped coil, (b) An equivalent circuit. Note that the use of a tapped coil increases the effective input impedance of the second amplifier stage.

## Amplifiers with Multiple Tuned Circuits:

The selectivity achieved with the single-tuned circuit of below figure is not sufficient in many applications-for instance, in the IF amplifier of a radio or a TV receiver. Greater selectivity is obtained by using additional tuned stages. Figure shows a BJT with tuned circuits at both the input and the output. In this circuit the bias details are shown, from which we note that biasing is quite similar to the classical arrangement employed in low-frequency discrete-circuit design. However, to avoid the loading effect of the bias resistors $\mathrm{R}_{\mathrm{m}}$ and $\mathrm{RB}_{2}$ on the input tuned circuit, a radio-frequency choke (RFC) is inserted in series with each resistor. Such chokes have high impedances at the frequencies of interest. The use of RFCs in biasing tuned RF amplifiers is common practice.

The analysis and design of the double-tuned amplifier of Fig is complicated by the Miller effect 12 due to capacitance Since the load is not simply resistive, as was s the case in the amplifiers, the Miller impedance e at the input will be complex. This reflected impedance will cause detuning of the input circuit as well as "skewing" of the response of the input circuit. Needless to say, the coupling introduced by $\mathrm{C} \mu$ makes tuning (or aligning) the amplifier quite difficult. Worse e still, the capacitor $\mathrm{C} \mu$ can cause oscillations to occur and methods exist for neutralizing the effect of $\mathrm{C} \mu$ using additional circuits arranged to feed back a current equal and opposite to that through $\mathrm{C} \mu$. An alternative, and preferred, approach is to use circuit configurations that do not suffer from the Miller effect. These are discussed later. Before leaving this section, however, we wish to point out that circuits of the type shown in Fig. 12.43 are usually designed utilizing the y-parameter mode 1 of the BJT. This is done because here, in view of the fact that C M plays a significant role, the y-parameter mode 1 makes s the analysis simpler (in comparison to that using the hybrid- $\pi$ model). Also, the y
parameters can easily be measured at the particular frequency of interest $\mathrm{W}_{0}$. For narrow-band amplifiers, the assumption is usually mad e that the y parameters remain approximately constant over the pass band.


A BJT amplifier with tuned circuits at the input and the output.

## The Cascode and the CC-CB Cascade:

We know that two amplifier configurations do not suffer from the Miller effect. These e are the cascode configuration and the common-collector common-bas e cascade. Figure shows tuned amplifiers based on these two configurations. The CC-CB cascade is usually preferred in IC implementations because its differential structure makes s it suitable for IC biasing techniques. (Note that the biasing details of the cascode circuit are not shown in Fig.

(a)

(b)

Two tuned-amplifier configurations that do not suffer from the Miller effect: (a) cascode and (b) common-collector common-base cascade.

## Synchronous Tuning:

In the design of a tuned amplifier with multiple tuned circuits the question of the frequency to which each circuit should be tuned arises. The objective, of course, is for the overall response to exhibit high pass band flatness and skirt selectivity. To investigate this question, we shall assume that the overall response is the product of the individual responses: in other words, that the stages do not interact.

Consider first the case of N identical resonant circuits, known as the synchronously tuned case. Figure8 shows the response of an individual stage and that of the cascade. Observe the bandwidth "shrinkage" of the overall response. The 3-dB bandwidth B of the overall amplifier is related to that of the individual tuned circuits, $\mathrm{W}_{0} / \mathrm{Q}$ is

$$
B=\frac{\omega_{0}}{Q} \sqrt{2^{1 / N}-1} \text { where } \sqrt{2^{1 / N}-1} \text { is known as the bandwidth-shrinkage factor. }
$$



Frequency response of a synchronously tuned amplifier.

## Stagger-Tuning:

A much better overall response is obtained by stagger-tuning the individual stages, as illustrated in Fig.. Stagger-tuned amplifiers are usually designed so that the overall response
exhibits maximal flatness around the center frequency $f_{0}$. Such a response can $b$ e obtained by transforming the response of a maximally flat (Butterworth) low-pass filter up the frequency axis to $\mathrm{W}_{0}$. We show here how this can be done. The transfer function of a second-order band pass filter can be expressed in terms of its poles as

$$
T(s)=\frac{a_{1} s}{\left(s+\frac{\omega_{0}}{2 Q}-j \omega_{0} \sqrt{1-\frac{1}{4 Q^{2}}}\right)\left(s+\frac{\omega_{0}}{2 Q}+j \omega_{0} \sqrt{1-\frac{1}{4 Q^{2}}}\right)}
$$



Stagger-tuning the individual resonant circuits can result in an overall response with a pass band flatter than that obtained with synchronous tuning

## UNIT-IV

Assignment-Cum-Tutorial Questions

## A. Objective Questions

1) An op-amp is a $\qquad$ amplifier
A) High gain differential
B) low gain differential
C) Moderate gain differential
D) unity gain
2) Draw the schematic symbol of an op-amp.
3) Draw the equivalent symbol of an op-amp and indicate all the parameters.
4) The common-mode rejection ratio of an ideal op-amp is [ ]
A) very high
B) Infinity
C) always unity
D)
unpredictable
5) The differential gain of an op-amp is
A) very high
B) very low
C) dependent on input voltage D )
about 100
6) The total output offset voltage is given by $\qquad$
7) The output of a particular op-amp increases 8 V in $12 \mu \mathrm{~s}$. The slew rate is [ ]
A) $90 \mathrm{~V} / \mu \mathrm{s}$
B) $0.67 \mathrm{~V} / \mu \mathrm{s}$
C) $1.5 \mathrm{~V} / \mu \mathrm{s}$
D) none of

## these

9) The input offset current equals the
A) difference between two base currents
B) average of two base currents
C) collector current divided by current gain
D) none of these
10) The output resistance of an ideal operational amplifier is [ ]
A) Infinity
B) Low
C) Very low
D) Zero

## 11. A tuned amplifier uses Load

12.For frequencies above the resonant frequency, a parallel LC circuit behaves as a ............. load

## 13.For frequencies below resonant frequency, a series $L C$ circuit behaves as a load

14. What is the relation between Bandwidth of synchronous tuned amplifier and individual tuned amplifier?
15.Neutralizing capacitor is connected between input and output nodes, to cancel out the effect of $\qquad$ Feedback
15. What is the reason for instability in Tuned amplifiers?
16. The selectivity of the tuned amplifier increases with Q factor
17. What is the relation between Q and series internal resistance of an inductor $\mathrm{r}_{\mathrm{s}}$.

## B. Subjective Questions

1. List out the ideal and practical specifications of an op-amp?
2. Define the following parameters with respect to op-amp A)Input bias current B) Input offset current C) Input offset voltage D) Slew rate
3. Identify why the input stage of an op-amp is DIBO differential amplifier.
4. Identify what can be the maximum gain of op-amp with the slew rate of $0.3 \mathrm{~V} / \mu \mathrm{s}$ when the input signal to op-amp is $0.03 \sin 2 \times 10^{5} \mathrm{t}$.
5. If the response to a square wave input, the output of an op-amp changes from -12 v to +12 v over a time interval of $0.5 \mu$ s identify the slew of the opamp?
6. How fast can the output of op-amp change by 15 v if its slew rate is $0.5 \mathrm{v} / \mu \mathrm{s}$ ?
7. Explain the inverting configuration under i) The closed loop gain ii)Effect of finite open loop gain iii) input and output resistance
8. Explain the Non inverting configuration under i) The closed loop gain ii)Effect of finite open loop gain
9. Briefly explain how op-amp will act as the Voltage follower
10. Explain the basic principle of Tuned Amplifiers.
11. Derive the expression for centre frequency and quality factor of a tuned Amplifier.
12. Explain about different types of tuned Amplifiers.
13. Discuss about the stability techniques in tuned amplifiers.
14. Explain briefly about Synchronous and stagger tuning

## C. Previous GATE Questions

1) The ideal OP-AMP has the following characteristics

GATE 2001
a. $\mathrm{R}_{\mathrm{i}}=\infty, \mathrm{A}_{\mathrm{V}}=\infty, \mathrm{R}_{0}=0$
b. $R_{i}=0, A_{V}=\infty, R_{o}=0$
c. $\mathrm{R}_{\mathrm{i}}=\infty, \mathrm{A}_{\mathrm{V}}=\infty, \mathrm{R}_{0}=\infty$
d. $\mathrm{R}_{\mathrm{i}}=0, \mathrm{~A}_{\mathrm{V}}=\infty, \mathrm{R}_{\mathrm{o}}=\infty$

## Answer: A

2) 

An ideal OP-AMP is an ideal
a. Voltage Controlled Current Source
b. Voltage Controlled Voltage Source
c. Current Controlled Current Source
d. Current Controlled Voltage Source

Answer: B
GATE 2004
3) If the differential voltage gain and the common mode voltage gain of a differential amplifier are 48 dB and 2 dB respectively, then common mode rejection ratio is
a. 23 dB
b. 25 dB
c. 46 dB
d. 50 dB

Answer: C

1) An amplifier using an op-amp with slewrate $1 \mathrm{v} / \mu \mathrm{s}$ has a gain of 40 Db . If this amplifier has to faithfully amplify sinusoidal signals from dc to 20 KHz without any slewrate introduced distortion, then the input signal level must not exceed----
a) 795 mv
b) 395 mv
c) 79.5 mv
d) 39.5 mv
2) A 741 opamp has again banwidth produt of 1 MHz . A non-inverting amplifier using this op-amp and having a voltage gain of 20 dB will exhibit a -3 dB bandwidth of -------
a) 50 KHz
b) 100 KHz
c) $100 / 17 \mathrm{KHz}$
d) $1000 / 7.07 \mathrm{KHz}$

## UNIT-V <br> FEEDBACK AMPLIFIERS

The concept of feedback involves feeding a small fraction of output at input by providing a connection from output to input. It is useful in several different ways in the design of amplifiers like stabilizing the gain, adjusting their impedances etc. and even to convert them into oscillators.

A classification of amplifiers, along with their ideal impedance characteristics is given. The concept of feedback, both positive and negative, is introduced and its impact on gain of the amplier is investigated. Different types of sampling and mixing are introduced along with their impact on the impedances of the amplifiers. Several amplifier circuits which use feedback for benefit are given.

### 5.1. Classification of amplifiers

There are several amplifiers and they are classified in different ways, depending upon different basis. According to one classification, amplifiers are divided into four types, namely, voltage amplifiers, current amplifiers, transresistance amplifiers and transconductance amplifiers. This classification is based on magnitudes of input and output impedances of amplifier relative to the source and load impedances, respectively.

The equivalent circuits of these four types of amplifiers, called models, are given and explained. These models use special kinds of sources, called dependent or controlled voltage and current sources. Each one is described briefly hereunder.

Voltage amplifier: By definition, voltage amplifier is one which provides a voltage output, $V_{o}$ that is proportional to source voltage, $V_{s}$ i.e.
$V_{o}=A_{v} V_{s}$
The proportionality factor, called voltage gain and indicated by $A_{v}$, should be independent of the magnitudes of the source and load resistances, $R_{s}$ and $R_{L}$, respectively. But in practice $A_{\nu}$ is dependent on $R_{s}$ and $R_{L}$, and hence the gain provided by the amplifier varies with source and load conditions. Mathematically, the amplifiers operation can be described by,

$$
\begin{equation*}
V_{o} \approx A_{v} V_{i} \approx A_{v} V_{s} \tag{5.2}
\end{equation*}
$$

Here, $V_{i}$ is input voltage. One can notice that the voltage amplifier model uses voltage controlled voltage source. The model circuit of this amplifier is shown in Figure 5.1(a). For the voltage amplifier to be ideal i.e. to have $A_{v}$ independent of $R_{s}$ and $R_{L}$, its input resistance $R_{i}$ should be infinite and output resistance $R_{o}$ should be zero. In practice, infinite or zero values for these resistances are not realizable. However, by setting $R_{i} \gg R_{s}$ and $R_{o} \ll R_{L}$ the behavior of voltage amplifier an be made approaching that of an ideal one.
Current amplifier: Current amplifier, by definition, is one which provides a load current $I_{L}$ that is proportional to source current $I_{s}$ i.e.
$I_{L}=A_{i} I_{s}$
The proportionality factor, called current gain and indicated by $A_{i}$, should be independent of the magnitudes of the source and load resistances, $R_{s}$ and $R_{L}$, respectively. The model circuit of this amplifier is shown in Figure 5.1(b). But in practice $A_{i}$ is dependent on $R_{s}$ and $R_{L}$, and hence the gain provided by the amplifier varies with source and load conditions. Mathematically, the amplifiers operation can be described by,

$$
\begin{equation*}
I_{L} \approx A_{i} I_{i} \approx A_{i} I_{s} \tag{5.4}
\end{equation*}
$$

Here, $I_{i}$ is input current. One can notice that the current amplifier model uses current controlled current source. For current amplifier to be ideal i.e. to have $A_{i}$ independent of $R_{s}$ and $R_{L}$, its input resistance $R_{i}$ should be zero and output resistance $R_{o}$ should be infinity. In practice, zero or infinite values for these resistances are not realizable, and hence, an ideal amplifier can never be realiezed. However, by setting $R_{i} \ll R_{s}$ and $R_{o} \gg R_{L}$, the behavior of current amplifier can be made approaching that of an ideal one.


Figure 5.1 Equivalent circuit of (a) voltage amplifier, (b) current amplifier, (c) transconductance amplifier and (d) transresistance amplifier

Transconductance amplifier: By definition, transconductance amplifier is one which provides a load current $I_{L}$ that is proportional to source voltage $V_{s}$ i.e.
$I_{L}=G_{m} V_{s}$
The proportionality factor, called transconductance and indicated by $G_{m}$, should be independent of the magnitudes of the source and load resistances, $R_{s}$ and $R_{L}$, respectively. The model circuit of this amplifier is shown in Figure 5.1(c). In practice $G_{m}$ is dependent on $R_{s}$ and $R_{L}$, and hence, transconductance provided by the amplifier varies with source and load conditions. Mathematically, the amplifiers operation can be described by,

$$
\begin{equation*}
I_{L} \approx G_{m} V_{i} \approx G_{m} V_{s} \tag{5.6}
\end{equation*}
$$

Here, $V_{i}$ is input voltage. One can notice that transconductance amplifier model uses voltage controlled current source. For transconductance amplifier to be ideal i.e. to have $G_{m}$ independent of $R_{s}$ and $R_{L}$, its input resistance $R_{i}$ as well output resistance $R_{o}$ should be infinity. In practice, infinite values for these resistances are not realizable. However, by setting $R_{i} \gg R_{s}$ and $R_{o} \gg R_{L}$ the behavior of transconductance amplifier can be made approaching that of an ideal one.
Transresistance amplifier: By definition, transresistance amplifier is one which provides a voltage output $V_{o}$ that is proportional to source current $I_{s}$ i.e.
$V_{o}=R_{m} I_{s}$
The proportionality factor, called transresistance and indicated by $R_{m}$, should be independent of the magnitudes of the source and load resistances, $R_{s}$ and $R_{L}$, respectively. The model circuit of this amplifier is shown in Figure 5.1(d). In practice $R_{m}$ is dependent on $R_{s}$ and $R_{L}$, and hence, transresistance provided by the amplifier varies with source and load conditions. Mathematically, the amplifiers operation can be described by,

$$
\begin{equation*}
V_{o} \approx R_{m} I_{i} \approx R_{m} I_{s} \tag{5.8}
\end{equation*}
$$

Here, $I_{i}$ is input current. One can notice that transresistance amplifier model uses voltage controlled current source. For transresistance amplifier to be ideal i.e. to have $R_{m}$ independent of $R_{s}$ and $R_{L}$, its input resistance $R_{i}$ as well output resistance $R_{o}$ should be zero. In practice, zero values for these
resistances are not realizable. However, by setting $R_{i} \ll R_{s}$ and $R_{o} \ll R_{L}$ the behavior of transresistance amplifier can be made approaching that of an ideal one.

Table 1 Ideal characteristics of amplifiers.

| S.No | Amplifier <br> attribute | Voltage <br> amplifier | Current <br> amplifier | Transconductance <br> amplifier | Transresistance <br> amplifier |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 1. | $R_{i}$ | $\infty$ | 0 | $\infty$ | 0 |
| 2. | $R_{o}$ | 0 | $\infty$ | $\infty$ | 0 |
| 3. | Transfer <br> characteristic | $V_{o}=A_{\nu} V_{s}$ | $I_{L}=A_{i} I_{s}$ | $I_{L}=G_{m} V_{s}$ | $V_{o}=R_{m} I_{s}$ |

It can be observed, in general, that the input and output resistances require to be high and low for voltage amplifiers, low and high for current amplifiers, low and low for transresistance amplifiers, high and high for transconductance amplifiers. Otherwise, the transfer gain of the amplifier becomes dependent upon the source as well as the load resistances. For example, the gain of the voltage amplifier can be expressed as,

$$
\begin{equation*}
A_{V S}=\frac{V_{o}}{V_{s}}=\frac{A_{v} V_{i} R_{L}}{\left(R_{o}+R_{L}\right) V_{s}}=A_{v} \frac{R_{L}}{\left(R_{o}+R_{L}\right)} \frac{R_{i}}{\left(R_{s}+R_{i}\right)} \tag{5.9}
\end{equation*}
$$

It can be noticed from the above expression that the voltage gain is dependent upon the source as well as the load resistances. Hence, this amplifier offers different amounts of gain in different circuits, depending upon the prevailing source and load resistances, which is really a drawback. It can also be observed that voltage gain becomes independent of source resistance if the input resistance is infinity or comparatively large with respect to source resistance. Similarly, the gain becomes independent of load resistance if the output resistance is zero, or comparatively small with respect to load resistance. Therefore, it is sufficient for a voltage amplifier to have high input and low output resistances for the sake of stabilized or fixed voltage gain. With similar reasoning applied to other types of amplifiers one can find out the requirements on their input and output resistances for giving fixed gains.

Applying feedback in proper configuration, it is possible to set the input and output resistances of the amplifiers at the required level, and have fixed amount of gain. However, one has to pay a price for the benefit of stabilized gain: the maximum realizable gain from the amplifier comes down.

### 5.2. Single loop feed back amplifier

Feedback is a technique in which the output signal is brought to the input side, mixed with the external signal and then the combined signal is given as input to the basic amplifier. Two types of feedback systems are there: widely used negative feedback, which reduces the gain when applied to amplifiers, and positive feedback used in the design of oscillators. With the application of negative feedback to amplifiers, two important changes take place.

1. The transfer gain gets desensitized i.e. dependence on the active circuit parameters, temperature and aging gets decreased, a desirable feature.
2. The transfer gain gets decreased, always unwanted.


Figure 5.2 Illustrating its basic components of single-loop feedback system.
A single-loop feedback system, shown in Figure 5.2, consists of basically the following elements:

1. Signal source: the signal source can be represented either by Thevenin's or by Norton's representation. But it convenient for manipulation to denote the source by Thevenin's voltage
source when the mixing is in series and by Norton's current source when the mixing is in shunt with the source.
2. Basic amplifier: It may belong to one of the four categories mentioned above. We go to feedback if its input or output resistance or both are not according to requirement. As an example we can consider BJT CB voltage amplifier whose input and output resistances are required to be high and low whereas in practice they are low and high. By applying appropriate feedback(voltage series) it is possible to adjust the input and output resistance values of the above amplifier to the desired levels.
3. Feedback network: It is usually a passive two-port network consisting of resistors, inductors and capacitors. So its transfer function or feedback factor $\beta$ is in general function of frequency, $f$ i.e. $\beta(f)$. In case of voltage sampling its input is voltage and in the case of current sampling it is current. For series mixing its output voltage is taken as feedback signal and in case of shunt mixing its output current is taken as feedback signal.
Comparator or mixer: The input resistance of an amplifier can be varied by selecting appropriate mixer circuit. Basically there are two types of mixers: series mixers and shunt mixers, as shown in Figure 5.3. We explain about each one of them and discuss their applications.

(a)

(b)

Figure 5.3 Input side feedback connections of a basic amplifier (a) Series comparion and (b) shunt comparison.
(i) Series or loop mixer: In this type of mixing, the feedback signal is connected in series with the input signal source. The feedback signal is necessarily a voltage and this type of mixing, you are going to see, increases the input resistance of the amplifier.
As shown in Figure 5.4(a), The input resistance of the amplifier in the absence of the feedback is

$$
\begin{equation*}
R_{i}=\frac{V_{s}}{I_{i}}=\frac{V_{i}}{I_{i}} \tag{5.10}
\end{equation*}
$$

The input resistance of the amplifier with feedback is

$$
\begin{equation*}
R_{i f}=\frac{V_{s}}{I_{i}}=\frac{V_{i}+V_{f}}{I_{i}} \tag{5.11}
\end{equation*}
$$

The denominators of the both the quantities are same. However, numerators are not same and in the expression of resistance with feedback it is large. Now, it is obvious that the series mixing increases the input resistance of the amplifier.


Figure 5.4 Input resistance affected by mixer connections.(a) Series connection and (b) shunt connection.
(ii) Shunt or node mixer: In this type of mixing, shown in Figure 5.3(b), the feedback signal is connected in shunt with the input signal source. The feedback signal is necessarily a current and this type of mixing decreases the input resistance of the amplifier.

As shown in Figure 5.4(b), the input resistance of the amplifier in the absence of the feedback is

$$
\begin{equation*}
R_{i}=\frac{V_{s}}{I_{i}}=\frac{V_{i}}{I_{i}} \tag{5.12}
\end{equation*}
$$

The input resistance of the amplifier with feedback is

$$
\begin{equation*}
R_{i f}=\frac{V_{i}}{I_{s}}=\frac{V_{i}}{I_{i}+I_{f}} \tag{5.13}
\end{equation*}
$$

The numerators of the both the quantities are same. However, denominators are not same and in the expression of resistance with feedback, it is large. Therefore, we can conclude that the shunt mixing decreases the input resistance of the amplifier.

(a)

(b)

Figure 5.5 Output side feedback connections of a basic amplifier (a) Voltage sampling and (b) current sampling.
4. Sampler: The output resistance of an amplifier can be varied by selecting appropriate sampler circuit. Basically there are two types of samplers: voltage samplers and current samplers, as shown in Figure 5.5. We discuss about both the types of samplers here and try to understand their impact on the out put resistances of the amplifiers.
(i) Voltage or node sampling: In this type of sampling the sampled signal is the voltage across the load. So the sampled signal is necessarily a voltage and this type of sampling decreases the output resistance of the amplifier. The output resistance of the amplifier in the absence of the feedback is

$$
\begin{equation*}
R_{o}=\frac{V}{I} \tag{5.14}
\end{equation*}
$$

The output resistance of the amplifier with feedback is

$$
\begin{equation*}
R_{o f}=\frac{V}{I+I_{f}^{\prime}} \tag{5.15}
\end{equation*}
$$

The numerators of the both the quantities are same. However, denominators are not same and in the expression of resistance with feedback, it is large. It is obvious that the voltage sampling decreases the output resistance of the amplifier.
(ii) Current or loop sampling: In this type of sampling the current through the load is sampled. So in this case the sampled signal is current and this type of sampling is useful to increase the output resistance of the amplifier. The output resistance of the amplifier in the absence of the feedback is

$$
\begin{equation*}
R_{o}=\frac{V}{I} \tag{5.16}
\end{equation*}
$$

The output resistance of the amplifier with feedback is

$$
\begin{equation*}
R_{o f}=\frac{V+V_{f}^{\prime}}{I} \tag{5.17}
\end{equation*}
$$

The denominators of the both the quantities are same. However, numerators are not same and in the expression of resistance with feedback it is large. Therefore it is obvious that the current sampling increases the input resistance of the amplifier.

### 5.3. Feedback specifications

Here is given frequently used terminology in feedback theory along with their meanings, definitions and expressions.

1. Loop gain or return ratio: The product $-\beta A$ is called the loop gain or return ratio.
2. Return difference or de-sensitivity The difference between unity and the loop gain is called the return difference. $D=1+\beta A$.
3. Amount of feedback The amount of feedback in decibels is
$N=\mathrm{dB}$ of feedback=20log $\left|\frac{A_{f}}{A}\right|=20 \log \left|\frac{1}{1+A \beta}\right|$.
For negative feedback, it is a negative number.


Figure 5.6 Illustrating schematic representation of a single-loop feedback system.

## 4. Transfer gain with negative feedback :

With the application of the negative feedback, the gain of the amplifier decreases. An expression for the reduced gain can be derived as follows: With reference to Figure 5.6, the output signal can be written that,

$$
\begin{aligned}
& X_{o}=A X_{i}=A X_{d}=A\left(X_{s}-X_{f}\right)=A\left(X_{s}-\beta X_{o}\right) \\
& X_{o}+\beta A X_{o}=A X_{s} \text { or } X_{o}(1+\beta A)=A X_{s}
\end{aligned}
$$

The transfer gain of the amplifier with feedback by definition then is

$$
\begin{equation*}
A_{f} \equiv \frac{X_{o}}{X_{s}}=\frac{A}{(1+\beta A)} \tag{5.19}
\end{equation*}
$$

One can easily notice that the gain of amplifier, with feedback, is less than that without feedback. Also this relation is independent of type of mixing as well as sampling i.e. it holds good for all the four types of configurations.
While deriving the expression above, the following fundamental assumptions of feedback amplifiers are made:

- The input signal is transmitted to the output through the amplifier $A$ and not through the feedback network.
- The output signal is transmitted to the input through the $\beta$ network and not through the amplifier, $A$.
- The reverse transmission factor $\beta$ of the feedback network is independent of the load and the source resistances $R_{L}$ and $R_{s}$.

5. Sensitivity It is defined as the fractional change in amplification with feedback $d A_{f} / A_{f}$ divided by the fractional change without feed back $d A / A$ is called the sensitivity of the transfer gain. Differentiating the expression for gain from Eq. (5.19), it can be found that,
$A_{f}=\frac{A}{(1+\beta A)} \rightarrow d A_{f}=\frac{d A}{(1+\beta A)}-\frac{\beta A d A}{(1+\beta A)^{2}}$
$d A_{f}=d A\left[\frac{1}{(1+\beta A)}-\frac{\beta A}{(1+\beta A)^{2}}\right]=d A\left[\frac{(1+\beta A)-\beta A}{(1+\beta A)^{2}}\right]=\frac{d A}{(1+\beta A)^{2}}$
$\frac{d A_{f}}{A_{f}}=\frac{d A}{(1+\beta A)^{2}} \frac{(1+\beta A)}{A}=\frac{1}{(1+\beta A)} \frac{d A}{A}$
Therefore, the sensitivity is
$\frac{d A_{f} / A_{f}}{d A / A}=\frac{1}{(1+\beta A)}$

### 5.4. Negative feedback

When any increase in the output signal results in a feedback signal into the input in such a way as to cause a decrease in the output signal, the amplifier is said to have negative feedback.
Advantages: When applied to amplifiers, the negative feedback has the following two primary applications.

- Any of the four basic types of amplifiers may be improved by the proper use of the negative feedback.
- Negative feedback improves the frequency response and linearity of the amplifiers significantly. Disadvantages: The amplifiers suffer on the following two conunts because of negative feedback.
- The gain of the amplifier gets reduced with the application of negative feedback.
- Negative feedback may result in instability and oscillations in the system.


Figure 5.7 Topologies of feedback amplifiers, considering the source resistance as part of the amplifier. (a) Voltage-series feedback applied to voltage amplifier (b) current-series feedback applied to transconductance amplifier (c) Current-shunt feedback applied to current amplifier (d) Voltage-shunt feedback applied to transresistance amplifier.

General characteristics of negative feedback
Different topologies of feedback amplifiers are shown in Figure 5.7. Here, the characteristics and impact of negative feedback on amplifiers are considered and explained.

## 1. Transfer gain with negative feedback

It has been found earlier an expression for gain of feedback amplifier in terms of gain without feedback. From the expression, it has also been noticed that the gain of amplifier, with feedback, is less than that without feedback, independent of type of mixing as well as sampling.
2. De-sensitivity of transfer amplification If the feedback network contains only stable passive elements, a substantial improvement in stability can be achieved.
Voltage- series feedback stabilizes voltage gain, $A_{V f}$
Current- series feedback stabilizes transconductance, $G_{m f}$
Current-shunt feedback stabilizes current gain, $A_{\text {If }}$
Voltage-shunt feedback stabilizes transresistance, $R_{M f}$
Table 5.2 Impact of negative feedback on amplifier performance.

| S.No. | Attribute | Voltage- series <br> feedback | Current- <br> series <br> feedback | Current- <br> shunt <br> feedback | Voltage- <br> shunt <br> feedback |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | $R_{\text {of }}$ | Decrease | Increase | Increase | Decrease |


| 2. | $R_{\text {if }}$ | Increase | Increase | Decrease | Decrease |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3. | Improved <br> characteristic | Voltage amplifier | Transconduct <br> ance amplifier | Current <br> amplifier | Transresistan <br> ce amplifier |
| 4. | Desensitizes | $A_{V f}$ | $G_{M f}$ | $A_{\text {If }}$ | $R_{M f}$ |
| 5. | Bandwidth | Increase | Increase | Increase | Increase |
| 6. | Nonlinear <br> distortion | Decrease | Decrease | Decrease | Decrease |

7. Frequency distortion If the feedback network does not contain reactive elements, the overall gain is not a function of frequency. Under these circumstances a substantial reduction in frequency and phase distortion is obtained.
8. Non linear distortion The harmonics generated due to nonlinearity in the amplifier gets divided by factor $D$ if the feedback is employed.
9. Reduction of noise It can be shown that the noise introduced in an amplifier is divided by the factor $D$ if the feedback is employed. If the additional gain required to compensate what is lost because of the presence of inverse feedback can be obtained by a readjustment of the circuit parameters rather than by the addition of extra stage, a definite reduction will result from the presence of the feedback. In particular, the hum introduced into the circuit by a poorly filtered power supply may be decreased appreciably.


Figure 5.8 Illustraing the decrease in voltage gain and increae in bandwidth for an amplifier when voltage-series feedback is applied.
10. Bandwidth improvement The bandwidth increases by a factor which is equal to the factor by which the gain decreases, as shown in Figure 5.8. The midband amplification with feedback $A_{\text {of }}$ equals to the midband amplification without feedback $A_{o}$ divided by $1+\beta A_{o}$. The high 3 dB frequency with feedback $f_{H f}$ equals the corresponding 3 dB frequency without feedback $f_{H}$ multiplied by the same factor $1+\beta A_{o}$. Also the low 3 dB frequency with feedback $f_{L f}$ equals the corresponding 3 dB frequency without feedback $f_{L}$ decreased by the same factor $1+\beta A_{o}$. For an audio or video amplifier, $f_{H} \gg f_{L}$ and hence the band width is $f_{H}-f_{L} \approx f_{H}$. Under these circumstances, one can observe that the gain bandwidth product is the same with or without feedback.

### 5.5. Analysis of amplifiers with feedback

Here, the input resistance and low output resistances various amplifires with feedback applied are discussed and expressions for them are derived.

### 5.5.1 Voltage amplifier with feedback

For the sake of gain stabilization, that is to make the voltage gain independent of the source and load resistances, the voltage amplifier requires to have high input resistance and low output resistance. So the mixing should be of such a type which can increase the input resistance i.e. series whereas the sampling should be able to decrease the output resistance i.e. voltage. Therefore it is the voltage-series configuration that can be used with the feedback of the voltage amplifiers.

Figure 5.10 A voltage amplifier with voltage-series feedback applied.


Input resistance: By definition, the input resistance with feedback is,

$$
\begin{equation*}
R_{i f} \equiv \frac{V_{s}}{I_{i}} \tag{5.21}
\end{equation*}
$$

From the input circuit, the source voltage can be expressed as, $V_{s}=I_{i} R_{i}+V_{f}=I_{i} R_{i}+\beta V_{o}$
From the output circuit, the output voltage can be written that,

$$
\begin{equation*}
V_{o}=\frac{A_{v} V_{i} R_{L}}{R_{o}+R_{L}}=A_{V} I_{i} R_{i} \tag{5.22}
\end{equation*}
$$

where $A_{V}$ is voltge gain, given by,

$$
A_{V} \equiv \frac{V_{o}}{V_{i}}=\frac{A_{v} R_{L}}{R_{o}+R_{L}} .
$$

Combining Eqs. (5.22) and (5.23), the ratio of $V_{s}$ to $I_{i}$ that is $R_{i f}$ can be found as,

$$
\begin{equation*}
R_{i f}=\frac{V_{s}}{I_{i}}=R_{i}\left(1+\beta A_{V}\right) \tag{5.24}
\end{equation*}
$$

Note that, the voltage gain $A_{v}$ represents the open-circuit gain, where as the voltage gain $A_{V}$ is the gain after taking the load $R_{L}$ into account, both, without feedback. Hence, one is related to other through,

$$
\begin{equation*}
A_{v}=\lim _{R_{L} \rightarrow \infty} A_{V} \tag{5.25}
\end{equation*}
$$

Note: In the above analysis, the source resistance has not been taken into account. In the case of series mixing, the input resistance usually rises to high values and when compared to it, the source resistance becomes small. As a consequence, the presence or absence of the source resistance in the input circuit does not make any significant difference in the analysis of the amplifier and hence we neglected it.
Output resistance: The output resistance with feedback $R_{o f}$ is the resistance at the output terminals of the amplifier but with load $R_{L}$ disconnected. Finding the output resistance is a two-step procedure: in the first step remove the input signal source by with a short if it is voltage source and by a open circuit if it is a current source. Simultaneously set the output resistance to infinity by removing the terminating load $R_{L}$ from the circuit. In the second step apply a voltage $V$ across the output terminals and then find the current $I$ driven by it into the circuit. The ratio voltage $V$ to current $I$ gives the output resistance of the amplifier. Hence,
$R_{o f} \equiv \frac{V}{I}$
As $V_{s}=0$, the input voltage can be written as,
$V_{i}=-V_{f}=-\beta V$
Using the Eq.(5.27), the current into the amplifier, from output side, can be written as,
$I=\frac{V-A_{v} V_{i}}{R_{o}}=\frac{V+\beta A_{v} V}{R_{o}}$

Hence, from Eq.(5.28), the ratio of $V$ to $I$, that is output resistance $R_{o f}$ of the amplifier with feedback can be found as,

$$
\begin{equation*}
R_{o f} \equiv \frac{V}{I}=\frac{R_{0}}{\left(1+\beta A_{v}\right)} \tag{5.29}
\end{equation*}
$$

The output resistance with feedback $R^{\prime}{ }_{o f}$, which includes $R_{L}$ as part amplifier is given by $R_{o f}$ in parallel with $R_{L}$. Hence,

$$
\begin{equation*}
R_{o f}^{\prime}=R_{o f} \square R_{L}=\frac{R_{0}^{\prime}}{\left(1+\beta A_{V}\right)} \tag{5.30}
\end{equation*}
$$

where

$$
\begin{equation*}
R_{o}^{\prime}=R_{o} \square R_{L}=\frac{R_{o} R_{L}}{R_{o}+R_{L}} \tag{5.31}
\end{equation*}
$$

## Current amplifier with feedback:

For current gain stabilization, the input resistance requires to be low whereas the output resistance needs to be high in current amplifier. So the mixing and sampling should be of such a type which can decrease the input resistance and increase the output resistance. Therefore it is the currentshunt configuration that can be used with the feedback of the current amplifiers.


Figure 5.11 A current amplifier with current-shunt feedback applied.
Input resistance: From the input circuit of the amplifier, the source current can be written as, $I_{s}=I_{i}+I_{f}=I_{i}+\beta I_{o}$.
From the output circuit, the output current can be written as,
$I_{o}=\frac{A_{i} I_{i} R_{o}}{R_{o}+R_{L}}=A_{I} I_{i}$
where $A_{I}$ is current gain, given by,
$A_{I}=\frac{A_{i} R_{o}}{R_{o}+R_{L}}$
Using expression for output current available in Eq. (5.33) into Eq. (5.32), the source current can be expressed as,
$I_{s}=I_{i}+I_{f}=I_{i}\left(1+\beta A_{I}\right)$
Using expression for source current available in Eq. (5.34), the input resistance $R_{i f}$ with feedback can be found as,
$R_{i f} \equiv \frac{V_{i}}{I_{s}}=\frac{V_{i}}{\left(1+\beta A_{I}\right) I_{i}}=\frac{R_{i}}{\left(1+\beta A_{I}\right)}$
Note that the gain $A_{i}$ represents the short-circuit i.e. $R_{L}=0$ current gain, where as the gain $A_{I}$ is the current gain with the load $R_{L}$ connected, both, without feedback. Hence, $A_{i}=\lim _{R_{L} \rightarrow 0} A_{I}$

Note: In the above analysis, we have not taken into account the source resistance. In the case of shunt mixing, the input resistance usually falls to small values and when compared to it, the source resistance becomes very large. As a consequence, the presence or absence of the source resistance in
the input circuit does not make any significant difference in the analysis of the amplifier and hence we neglected it.
Output resistance: To find the output resistance, $R_{o f}$ remove the external signal source and termination resistance. Now apply a voltage $V$ across the output terminals and then find the current $I$ driven by it into the circuit. The ratio of voltage $V$ to current $I$ gives the output resistance of the amplifier. Now, from the output circuit current into amplifier can be written as,
$I=\frac{V}{R_{o}}-A_{i} I_{i}$
From the input circuit as $I_{s}=0$, the input current can be expressed as,
$I_{i}=-I_{f}=-\beta I_{o}=\beta I$.
Manipulating the relation available in Eq. (5.36), the ratio of $V$ to $I$ i.e.the output resistnace, $R_{o f}$ with feedback can be found as,
$I=\frac{V}{R_{o}}-\beta A_{i} I$
$I\left(1+\beta A_{i}\right)=\frac{V}{R_{o}}$
$R_{o f} \equiv \frac{V}{I}=R_{0}\left(1+\beta A_{i}\right)$
The output resistance with feedback, $R_{o f}^{\prime}$ which includes $R_{L}$ as part amplifier, is given by $R_{o f}$ in parallel with $R_{L}$.
$R_{o f}^{\prime}=R_{o f} \square R_{L}=\frac{R_{0}^{\prime}\left(1+\beta A_{i}\right)}{\left(1+\beta A_{I}\right)}$
where

$$
\begin{equation*}
R_{o}^{\prime}=R_{o} \square R_{L}=\frac{R_{o} R_{L}}{R_{o}+R_{L}} \tag{5.39}
\end{equation*}
$$

## Transconductance amplifier with feedback:

For the stabilization of transconductance, both the input and output resistances requires to be high. The type of mixing that can increase the input resistance is series and the type of sampling that can increase the output resistance is current sampling. So the configuration that is suitable for the feedback of the transconductance amplifiers is the current-series.
Input resistance: The input impedance of the amplifier with feedback is ratio of source voltage to input current. From the input circuit, the source voltage can be written as,
$V_{s}=I_{i} R_{i}+V_{f}=I_{i} R_{i}+\beta I_{o}$
But from the output circuit, the output current can be expressed as,
$I_{o}=\frac{G_{m} V_{i} R_{o}}{R_{o}+R_{L}}=\frac{G_{m} I_{i} R_{o} R_{i}}{R_{o}+R_{L}}$
Combining Eqs.(5.40) and (5.41), the source voltage can be found as,
$V_{s}=I_{i} R_{i}+\beta I_{o}=I_{i} R_{i}+\beta \frac{G_{m} I_{i} R_{o} R_{i}}{R_{o}+R_{L}}=I_{i} R_{i}\left(1+\frac{\beta G_{m} R_{o}}{R_{o}+R_{L}}\right)=I_{i} R_{i}\left(1+\beta G_{M}\right)$
From Eq. (5.42), the ratio $V_{s} / I_{i}$, that is the input resistance with feedback $R_{i f}$ can be found as,

$$
\begin{equation*}
R_{i f}=\frac{V_{s}}{I_{i}}=R_{i}\left(1+\beta G_{M}\right) \tag{5.43}
\end{equation*}
$$

where $G_{M}$ is transconductance given by,
$G_{M} \equiv \frac{I_{o}}{V_{i}}=\frac{G_{m} R_{o}}{R_{o}+R_{L}}$

Note that, whereas $G_{m}$ represents the short-circuit transconductance, the $G_{M}$ is the transconductance without feedback after taking the load $R_{L}$ into account. Thus,
$G_{m}=\lim _{R_{L} \rightarrow 0} G_{M}$


Figure 5.12 A transconductance amplifier with current-series feedback applied.
Output resistance: To find the output resistance with feedback remove the external signal source and replace the load resistance with a source of voltage $V$ across the output terminals and then find the current $I$ driven by it into the circuit. The ratio of voltage $V$ to current $I$ gives the output resistance of the amplifier. Now from the output circuit

$$
\begin{equation*}
I=\frac{V}{R_{o}}-G_{m} V_{i} \tag{5.46}
\end{equation*}
$$

In the input circuit, as $V_{s}=0$, the input voltage $V_{i}$ can be expressed as $V_{i}=-V_{f}=-\beta I_{o}=\beta I$.
Combining Eqs.(5.46) and (5.47) and then solving for output resistance, $R_{\text {of }}$ gives,
$I=\frac{V}{R_{o}}-\beta G_{m} I \rightarrow I\left(1+\beta G_{m}\right)=\frac{V}{R_{o}}$
$R_{o f} \equiv \frac{V}{I}=R_{0}\left(1+\beta G_{m}\right)$
The output resistance with feedback, $R^{\prime}{ }_{o f}$ which includes $R_{L}$ as part amplifier is given by $R_{o f}$ in parallel with $R_{L}$.

$$
\begin{equation*}
R_{o f}^{\prime}=R_{o f} \square R_{L}=\frac{R_{0}^{\prime}\left(1+\beta G_{m}\right)}{\left(1+\beta G_{M}\right)} \tag{5.49}
\end{equation*}
$$

where $R_{o}^{\prime}=R_{o} \square R_{L}$

## Transresistance amplifier with feedback:

For transresistance stabilization, the input as well as the output resistances of the amplifier requires to be low. So we require using the type of mixing and sampling which can decrease the input and output resistances. The suitable configuration naturally will be current shunt for the transresistance amplifiers.
Input resistance: From the input circuit, the source current can be written as,
$I_{s}=I_{i}+I_{f}=I_{i}+\beta V_{o}$.
And from the output circuit, the output voltage can be expressed as,

$$
\begin{equation*}
V_{o}=\frac{R_{m} I_{i} R_{L}}{R_{o}+R_{L}}=R_{M} I_{i} \tag{5.50}
\end{equation*}
$$

where $R_{M}$ is transresistance, given by,

$$
\begin{equation*}
R_{M}=\frac{R_{m} R_{L}}{R_{o}+R_{L}} \tag{5.52}
\end{equation*}
$$

Combining Eqs.(5.50) and (5.51), the source current can be written as, $I_{s}=I_{i}+\beta R_{M} I_{i}=I_{i}\left(1+\beta R_{M}\right)$
Solivng Eq. (5.53)for the ratio of input voltage to source current i.e. input resistance, results in,

$$
\begin{equation*}
R_{i f}=\frac{V_{i}}{I_{s}}=\frac{R_{i}}{\left(1+\beta R_{M}\right)}=\frac{V_{i}}{\left(1+\beta R_{M}\right) I_{i}}=\frac{R_{i}}{\left(1+\beta R_{M}\right)} \tag{5.53}
\end{equation*}
$$

where
$R_{M} \equiv \frac{V_{o}}{I_{i}}=\frac{R_{m} R_{L}}{R_{o}+R_{L}}$
Note that $R_{m}$ represents the open-circuit transresistance, whereas the $R_{M}$ is the transresistance when the termination is $R_{L}$.


Figure 5.13 A transresistance amplifier with voltage-shunt feedback applied.
Output resistance: The resistance $R_{\text {of }}$ is output resistance with feedback i.e. resistance seen when looking into output terminals without load $R_{L}$. If a voltage source $V$ drives a current $I$ into the output terminals of the amplifier in the absence of signal source and load resistance, then the output resistance becomes,

$$
\begin{equation*}
R_{o f} \equiv \frac{V}{I} . \tag{5.56}
\end{equation*}
$$

For Series comparison $R_{i f}>R_{i}$ and for Shunt comparison $R_{i f}<R_{i}$

| Series mixing | Shunt mixing |
| :--- | :--- |
| Voltage-series $R_{i f}=R_{i}\left(1+\beta A_{V}\right)$ | $R_{i f}=\frac{R_{i}}{\left(1+\beta R_{M}\right)}$ |
| Voltage-shunt |  |
| Current-series $R_{i f}=R_{i}\left(1+\beta G_{M}\right)$ | $R_{i f}=\frac{R_{i}}{\left(1+\beta A_{I}\right)}$ |

For Voltage sampling $R_{o f}<R_{o}$ and for Current sampling $R_{o f}>R_{o}$

| Voltage sampling | Currrent sampling |
| :---: | :--- |
| Voltage-series $R_{o f}^{\prime}=\frac{R_{0}^{\prime}}{\left(1+\beta A_{V}\right)}$ | Current-series $R_{o f}^{\prime}=\frac{R_{0}^{\prime}\left(1+\beta G_{m}\right)}{\left(1+\beta G_{M}\right)}$ |
| Voltage-shunt $R_{o f}^{\prime}=\frac{R_{0}^{\prime}}{\left(1+\beta R_{M}\right)}$ | Current-shunt $\quad R_{o f}^{\prime}=\frac{R_{0}^{\prime}\left(1+\beta A_{i}\right)}{\left(1+\beta A_{I}\right)}$ |

The current into the amplifier from the output terminals can be expressed as,
$I=\frac{V-R_{m} I_{i}}{R_{o}}$

From the input circuit, as $I_{s}=0$, the input current can be found as,
$I_{i}=-I_{f}=-\beta I_{o}=\beta I$.
Combining Eqs.(5.57) and (5.58), and then solving for $V$ to $I$ ratio, the output resistance can be established as,
$I=\frac{V+\beta R_{m} I}{R_{o}}$
$R_{o f} \equiv \frac{V}{I}=\frac{R_{0}}{\left(1+\beta R_{m}\right)}$
The output resistance, $R_{o f}^{\prime}$ with feedback including $R_{L}$ as part amplifier, is given by $R_{o f}$ in parallel with $R_{L}$.

$$
\begin{equation*}
R_{o f}^{\prime}=R_{o f} \square R_{L}=\frac{R_{0}^{\prime}}{\left(1+\beta R_{M}\right)} \tag{5.60}
\end{equation*}
$$

where $R_{o}^{\prime}=R_{o} \square R_{L}$.
Here $R_{o}^{\prime}$ is output resistance without feedback but taking the load $R_{L}$ into account. Therefore, $R_{m}=\lim _{R_{L} \rightarrow \infty} R_{M}$

### 5.6. Example circuits with feedback

### 5.6.1. Method of Analysis of a Feedback Amplifier

It is desirable to separate the feedback amplifier into two blocks, the basic amplifier A and the feedback network $\beta$, because with a knowledge of $A$ and $\beta$, we can calculate the important characteristics of the feedback system, namely, $A_{f}, R_{i f}$, and $R_{o f}$. The basic amplifier configuration without feedback but taking the loading of the $\beta$ network into account is obtained by applying the following rules:
To find the input circuit:

1. Set $V_{o}=0$ for voltage sampling. In other words, short the output node.
2. Set $I_{o}=0$ for current sampling. In other words, open the output loop.

To find the output circuit:

1. Set $V_{i}=0$ for shunt comparison. In other words, short the input node.
2. Set $I_{i}=0$ for series comparison. In other words, open the input loop.

These procedures ensure that the feedback is reduced to zero without altering the loading on the basic amplifier.

Table 5.2 Analysis of feedback amplifiers.

| S.No. | Type of feedback | Voltageseries | Currentseries | Currentshunt | Voltageshunt |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Attribute |  |  |  |  |
| 1. | Feedback signal | Voltage | Voltage | Current | Current |
| 2. | Sampled signal | Voltage | Current | Current | voltage |
| 3. | To find input loop set | $V_{o}=0$ | $V_{o}=0$ | $I_{0}=0$ | $I_{o}=0$ |
| 4. | To find output loop set | $V_{o}=0$ | $I_{0}=0$ | $I_{0}=0$ | $V_{o}=0$ |
| 5. | Signal source | Thevenin's | Thevenin's | Norton's | Norton's |
| 6. | Feedback factor, 8 | $V_{f} / V_{o}$ | $V_{f} / l_{0}$ | $I_{f} / I_{0}$ | $I_{f} / V_{o}$ |
| 7. | Gain, A | $A_{v}=V_{o} / V_{i}$ | $G_{M}=I_{0} / V_{i}$ | $A_{1}=I_{0} / I_{i}$ | $R_{M}=V_{0} / I_{i}$ |
| 8. | Desensitivity D | $1+6 A_{V}$ | $1+6 G_{M}$ | $1+6 A_{l}$ | $1+6 R_{M}$ |
| 9. | Gain with feedback, $A_{f}$ | $A_{v} / D$ | $G_{M} / D$ | $A_{1} / D$ | $R_{M} / D$ |
| 10. | $R_{\text {if }}$ | $R_{i} D$ | Ri $D$ | $R_{i} / D$ | $R_{i} / D$ |
| 11. | $R_{\text {of }}$ | $R_{o} /\left(1+6 A_{v}\right)$ | $R_{o}\left(1+8 G_{m}\right)$ | $R_{o}\left(1+6 A_{i}\right)$ | $R_{o} /\left(1+8 R_{m}\right)$ |
| 12. | $R_{\text {of }}^{\prime}=R_{o f}\| \| R_{L}$ | $R^{\prime}{ }_{0} D$ | $\begin{gathered} R_{o}^{\prime}\left(1+8 G_{m}\right) / \\ D \end{gathered}$ | $\begin{gathered} R_{o}^{\prime}\left(1+6 A_{i}\right) / \\ D \end{gathered}$ | $R^{\prime}{ }_{0} D$ |

e complete analysis of a feedback amplifier is obtained by carrying out the following steps:

1. Identity the topology. (a) Is the feedback signal $X_{f}$ a voltage or a current? In other words, is $X_{f}$ applied in series or in shunt with the external excitation? (b) Is the samples signal $X_{o}$ a voltage or a current? In other words, is the sampled signal taken at the output node or from the output loop?
2. Draw the basic amplifier circuit without feedback, following the rules listed above.
3. Use a Thevenin's source if $X_{f}$ is a voltage and a Norton's source of $X_{f}$ is a current.
4. Replace each active device by the proper model (for example, the hybrid $-\pi$ model for a transistor at high frequencies or the $h$ - parameter model at low frequencies).
5. Indicate $X_{f}$ and $X_{o}$ on the circuit obtained by carrying out steps 2,3 and 4. Evaluate $\beta$, using $\beta$ $=X_{f} / X_{o}$
6. Evaluate $A$ by applying KVL and KCL to the equivalent circuit obtained after step 4.
7. From $A$ and $\beta$, find $D, A_{f}, R_{i f}, R_{o f}$ and $R_{o f}^{\prime}$.

## The Stability Problem:

Transfer function of feedback amplifier

$$
A_{f}(s)=\frac{A(s)}{1+A(s) \beta(s)}
$$

For physical frequencies $s=j \omega$, Eq. (10.81) becomes

$$
A_{f}(j \omega)=\frac{A(j \omega)}{1+A(j \omega) \beta(j \omega)}
$$

Thus the loop gain $A(j \omega) \beta(j \omega)$ is a complex number that can be represented by its magnitude and phase,

$$
\begin{aligned}
L(j \omega) & \equiv A(j \omega) \beta(j \omega) \\
& =|A(j \omega) \beta(j \omega)| e^{j \phi(\omega)}
\end{aligned}
$$

## Effect of Feedback on the amplifier poles :

Stability and poles location



## Poles of the feedback amplifier :

From the closed-loop transfer function we see that the poles of the feedback amplifier are the zeros of $1+A(s) \beta(s)$. That is, the feedback-amplifier poles are obtained by solving the equation

$$
1+A(s) \beta(s)=0
$$

which is called the characteristic equation of the feedback loop. It should therefore be apparent that applying feedback to an amplifier changes its poles.

## Amplifier with single pole response

Consider first the case of an amplifier whose open-loop transfer function is characterized by a single pole:

$$
\begin{equation*}
A(s)=\frac{A_{0}}{1+s / \omega_{P}} \tag{10.86}
\end{equation*}
$$

The closed-loop transfer function is given by

$$
A_{f}(s)=\frac{A_{0} /\left(1+A_{0} \beta\right)}{1+s / \omega_{P}\left(1+A_{0} \beta\right)}
$$

Thus the feedback moves the pole along the negative real axis to a frequency $\omega_{P \rho}$,

$$
\omega_{P f}=\omega_{P}\left(1+A_{0} \beta\right)
$$



Figure 10.36 Effect of feedback on (a) the pole location and (b) the frequency response of an amplifier having a single-pole, open-loop response.

## Stability study using bode plots :

## Gain and phase margins

whether a feedback amplifier is or is not stable can be determined by examining its loop gain $A \beta$ as a function of frequency. One of the simplest and most effective means for doing this is through the use of a Bode plot for $A \beta$.
The difference between the value of at $\omega 180$ and unity, called the gain margin, is usually expressed in decibels. The gain margin represents the amount by which the loop gain can be increased while stability is maintained.
Feedback amplifiers are usually designed to have sufficient gain margin to allow for the inevitable changes in loop gain with temperature, time, and so on.

## OSCILLATORS

In the design of electronic systems, the need frequently arises for signals having prescribed standard waveforms (e.g., sinusoidal, square, triangle, pulse, etc). These waveforms are commonly used in computers, control systems, communication systems and test measurement systems.

There are two common ways for generating sinusoids:

1. Positive feedback loop with non-linear gain limiting
2. Appropriately shaping other waveforms such as a triangle waves.

## SINUSOIDAL OSCILLATORS:

Commonly referred to as linear sine-wave oscillators although some forms of non-linearity have to be employed to limit the output amplitude. Analysis of the circuits is more difficult as s-plane analysis cannot be directly applied to the non-linear part of the circuit. The basic structure of a sinusoidal oscillator consists of an amplifier and a frequency selective network connected in a positive feedback loop.


Figure 1: Basic structure of a sinusoidal oscillator.
A positive-feedback loop is formed by an amplifier and a frequency-selective network. In an actual oscillator circuit, no input signal will be present; here an input signal $\mathrm{x}_{\mathrm{s}}$ is employed to help explain the principle of operation. Note that the feedback signal $X_{F}$ is summed with a positive sign:

$$
\mathrm{A}_{\mathrm{f}}(\mathrm{~s})=\frac{\mathrm{A}(\mathrm{~s})}{1-\mathrm{A}(\mathrm{~s}) \beta(\mathrm{s})}
$$

The loop gain is:

$$
\mathrm{L}(\mathrm{~s})=\mathrm{A}(\mathrm{~s}) \beta(\mathrm{s})
$$

And the characteristic equation can be written as:

$$
1-\mathrm{L}(\mathrm{~s})=0
$$

If at a specific frequency $f_{0}$, the loop gain $A \beta$ is equal to unity, it follows that $A_{f}$ will be infinite. Such a circuit is by definition an oscillator.

Thus for the sinusoidal oscillator at $\omega_{0}$ :

$$
L(j \omega 0)=A(j \omega 0) \cdot \beta(j \omega 0)=1
$$

This condition is called Barkhausen Criteria for oscillation, in which:

## "UNITY GAIN, ZERO PHASE SHIFT"

It should be noted that the frequency of oscillation $\omega_{0}$ is determined by the phase characteristics of the feedback loop. The loop oscillates at the frequency for which the phase is ZERO. The steeper the phase shift as a function of frequency $\phi(\omega)$, the more stable the frequency of oscillation.


Fig. 12.2 Dependence of the oscillator frequency stability on the slope of the phase response. A steep phase response (that is, large $d \phi / d \omega$ ) results in a small $\Delta \omega_{0}$ for a given change in phase $\Delta \phi$ (resulting from a change in a circuit component).

## NON-LINEAR AMPLITUDE CONTROL:

Generally, it is difficult to design circuits with $\mathrm{A} \beta=1$ as circuit parameters vary with temperature, time, and component values.

$$
\begin{array}{lll}
\text { If } & A \beta<1 & \text { oscillator ceases, } \\
\text { If } & A \beta>1 & \text { oscillation grows until circuit saturates. }
\end{array}
$$

It is required to have a mechanism to force $\mathrm{A} \beta=1$. This is accomplished by employing a nonlinear circuit for gain control:

- Design circuit with $\mathrm{A} \beta>1$ as voltage of oscillation increases, gain control mechanism kicks in and reduces gain to 1 .
- Design circuit with right half plane poles. The gain control pulls the poles back to the imaginary axis.


## Two approaches:

1. The first approach uses a limiter circuit, oscillations are allowed to grow until the level reaches the limiter set value. Once the limiter comes into operation, the amplitude remains constant. The limiter should be designed to minimize non-linear distortion.
2. The second method uses a resistive element in the feedback loop whose resistance can be controlled by the sinusoidal output amplitude. Diodes or JFETs (operating in triode region) are commonly used.

A popular limiter circuit for amplitude control can be seen below:


## II. OPAMP - RC OSCILLATORS:

## 1. WIEN-BRIDGE OSCILLATOR



Wien Bridge oscillator without amplitude stabilization
The loop gain can be found by multiplying the transfer function of the feedback path, $\mathrm{V}_{\mathrm{a}}(\mathrm{s}) / \mathrm{V}_{0}(\mathrm{~s})$, by the amplifier gain.

$$
\begin{gathered}
L(s)=\left[1+\frac{R}{R_{2}}\right] \frac{Z P}{Z_{P}+Z} \\
L(j \omega)=\frac{1+\frac{R 1}{R 2}}{3+j\left(\omega R C-\frac{1}{\omega R C}\right)}
\end{gathered}
$$

The loop gain will be a real number (i.e., the phase will be zero) at one frequency $\omega_{0}$ given

$$
\begin{aligned}
& \omega 0 \mathrm{RC}= \frac{1}{\omega 0 \mathrm{RC}} \\
& \\
& \omega_{0}=\mathrm{RC}^{1}
\end{aligned}
$$

To obtain sustained oscillation at this frequency, the magnitude of the loop gain should be unity which can be achieved by setting:

$$
\begin{aligned}
& \underline{\mathrm{R}} \underline{2}=2 \\
& \mathrm{R} 1
\end{aligned}
$$

To ensure that oscillation starts, one chooses R2/R1 slightly greater than 2 .

The amplitude of the oscillation can be controlled using a non-linear limiter as seen below.


Fig. 12.5 A Wien-bridge oscillator with a limiter used for amplitude control.

## 2. PHASE SHIFT OSCILLATOR

Figure 12.7 shows the basic structure of the phase shift oscillator. It consists of a negative gain amplifier ( -K ) with a three-section ( $3^{\text {rd }}$ order) RC ladder network in the feedback.


Fig. 12.7 Phase-shift oscillator.

The circuit will oscillate at the frequency for which the phase shift of the RC network is $180^{\circ}$. Only at this frequency will the phase shift around the loop be $\mathbf{0}^{\mathbf{O}}\left(\mathbf{3 6 0}{ }^{\mathbf{O}}\right)$. Three RC sections are required to produce a $180^{\circ}$ phase shift at a finite frequency.

The value of $K$ is chosen to be slightly higher than the inverse of the magnitude of the RC network transfer function at the frequency of oscillation.


Fig. 12.8 Practical phase-shift oscillator with a limiter for amplitude stabilization.

## 3. ACTIVE FILTER TUNED OSCILLATOR

In this type of filter, Figure 12-10, a filter is used to select a particular frequency in the spectrum of a square wave (usually the fundamental frequency). Output of the filter is a sinewave and is taken as the output of the oscillator. The output is fed back to a limiter which is used to convert a sinewave to a squarewave. The squarewave signal then becomes the input of the filter.

The actual circuit is shown in Figure 10.11, the limiter is a pair of diodes to have a squarewave at v2. This filter is an active filter (we will study this filter later) to select the fundamental frequency and provides the output at v1.

The op-amp RC oscillator circuits are useful for operation in the $10 \mathrm{~Hz}-1 \mathrm{MHz}$ range due to limitations in passive component size (low frequency) and op-amp slew rate (high frequency). For higher frequencies, circuits that employ transistors together with LC tuned circuits or crystals are commonly used.


Fig. 12.10 Block diagram of the active-filter funed oscillator.


Fig. 12.11 Practical implementation of the active-filter tuned oscillator.

## III. LC AND CRYSTAL OSCILLATORS:

Oscillators utilizing transistors and LC tuned circuits or crystals are useful for operation in the range from 100 KHz to 500 MHz . They exhibit higher Q than RC types (more stable). However, LC oscillators are difficult to tune over wide range of frequency and crystal oscillator operates at a single frequency. The extremely stable response of the crystal oscillators has made them very popular, particularly for digital timing signals.

## LC TUNED OSCILLATOR

Two common used configurations are the Colpitts and the Hartley oscillators. The basic circuit structures without biasing can be seen below.


Fig. 12.12 Two commonly used configurations of LC-tuned oscillators: (a) Colpitts; (b) Hartley.

Both circuits utilize a parallel LC circuit connected between the collector and the base with a fraction of the tuned circuit voltage fed to the emitter of the transistor. The resistor R models the losses of the inductor, the load resistance of the oscillator and the output resistance of the transistor.

If the frequency of operation is sufficiently low, we can neglect the transistor parasitic capacitances. The frequency of oscillation is determined by the resonant frequency of the parallel tuned circuit (also known as a tank circuit). For the Colpitts oscillator:

For the Hartley oscillator

$$
\omega 0=\frac{1}{\frac{\mathrm{~L}\left(\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}\right)}{}}
$$

$$
\omega 0=\frac{1}{\mathrm{C}\left(\mathrm{~L}_{1}+\mathrm{L}_{2}\right)}
$$

The ratio $\mathrm{L}_{1} / \mathrm{L}_{2}$ or $\mathrm{C}_{1} / \mathrm{C}_{2}$ determines the feedback factor and thus must be adjusted in conjunction with the transistor gain to ensure that oscillations will start.

To determine the oscillation condition for the Colpitts oscillator, we replace the transistor with its equivalent circuit. To simplify the analysis, we neglect the transistor capacitances except capacitance Све is a part of C 2 .


Fig. 12.13 Equivalent circuit of the Colpitts oscillator of Fig. 12.12(a). To simplify the analysis, $\mathrm{C}_{\mu}$ and $r_{\pi}$ are neglected. We can consider $C_{m}$ to be part of $C_{2}$, and we can include $r_{0}$ in $R$.

A node equation at the transistor collector (C) yields:

$$
\mathrm{sC} \underset{2 \pi}{\mathrm{~V}}+\mathrm{g}_{\mathrm{m}} \underset{\pi}{\mathrm{~V}}+\left(\frac{1}{\mathrm{R}}+\mathrm{sC}\right)\left(1+\mathrm{s}_{1}^{2} \mathrm{LC} \underset{2}{ }\right) \mathrm{V}_{\pi}=0
$$

Since $\mathrm{V} \pi \neq 0$ (oscillations have started), it can be eliminated (i.e., the other terms are zero).

$$
\begin{aligned}
& s^{3} \mathrm{LC} 1 \mathrm{C} 2+\mathrm{s}^{2}\left(\mathrm{~L}^{\mathrm{C}} \mathrm{R}^{2}\right)+\mathrm{s}(\mathrm{C} 1+\mathrm{C} 2)+\left(\overline{\mathrm{gm}}+\mathrm{R}^{1}\right)=0 \\
& \left(\mathrm{~g}_{\mathrm{m}}+\frac{1}{\mathrm{R}}-\frac{\omega 2 \mathrm{CC}_{2}}{\mathrm{R}}\right)+\mathrm{j}\left[\omega\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right)-\omega^{3} \mathrm{LCC} \underset{12}{ }\right]=0
\end{aligned}
$$

## For oscillations to start, both the real and imaginary parts must be zero.

Setting the imaginary part to zero gives

$$
\omega 0=\frac{1}{\sqrt{\left.\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}+\mathrm{C}_{2}}\right)}}
$$

which is the resonant frequency of the tank circuit.
Setting the real part to zero yields

$$
\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}=\mathrm{gmR}
$$

For sustained oscillation, the magnitude of the gain from the base to collector $\left(g_{m} R\right)$ must be equal to the inverse of the voltage ratio provided by the capacitive divider:

$$
\frac{\mathrm{v}_{\mathrm{be}}}{\mathrm{v}_{\mathrm{ce}}}=\frac{\mathrm{C}_{1}}{\mathrm{C}_{2}}
$$

For oscillation to start, the loop gain must be greater than unity which is equivalent to

$$
\mathrm{gmR}>\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}
$$

As oscillation grows in amplitude, the transistors non-linear characteristics reduce the loop gain to unity, thus sustaining oscillations. An example of a complete Colpitts oscillator is shown below


The radio frequency choke (RFC) in this oscillator provides a high reactance at $\omega_{0}$ but a low DC resistance. Unlike the op-amp oscillators that incorporate special amplitude control circuitry, LC tuned oscillators utilize the non-linear $\mathrm{i}_{\mathrm{c}}$-vbe characteristics of the BJT (or $\mathrm{i}_{\mathrm{d}}$ versus $\mathrm{Vgs}^{\text {for }}$ FET) for amplitude control. As the oscillations grow, the effective gain of the transistor is reduced below its small signal value. The LC tuned oscillators are known as self-limiting oscillators.

Reliance on the non-linear characteristics of the BJT (or the FET) implies that the collector (drain) current waveform will be nonlinearity distorted. Nevertheless, sinusoidal of high purity because of the filtering action of the LC tuned circuit.

## CRYSTAL OSCILLATORS

A piezoelectric crystal, such as quartz, exhibits electro-mechanical resonant characteristics that are very stable (with time and temperature) and high selectivity (having very high Q factor). The circuit symbol of a crystal is shown below.

The resonant properties are characterized by a large inductance L (as high as hundreds of Henrys), a very small series capacitance $\mathrm{C}_{\mathrm{s}}$ (as small as 0.0005 pF ), a series resistance r representing a Q factor $\left(\mathrm{Q}=\omega_{0} \mathrm{~L} / \mathrm{r}\right.$ that can be as high as few hundred thousand) and a parallel capacitance $\mathrm{C}_{\mathrm{p}}$ (a few picoFarad).


Fig. 12.15 A piezoelectric crystal: (a) circuit symbol; (b) equivalent circuit; (c) crystal reactance versus frequency [note that, neglecting the small resistance $\left.r, Z_{\text {crstal }}=j X(\omega)\right]$.

Capacitance $C_{p}$ represents the electrostatic capacitance between the two parallel plates of the crystal ( $\mathrm{C}_{\mathrm{p}} \gg \mathrm{C}_{\mathrm{s}}$ ). Since the Q factor is so high, we can neglect the resistance r and express the crystal impedance as:
which can be manipulated to the form

$$
\mathrm{Z}(\mathrm{~s})=\frac{1}{\mathrm{sCp}+\frac{1}{\mathrm{sL}+1 / \mathrm{sCs}}}
$$

$$
Z(s)=\frac{1}{s C_{p ~ s}} \frac{s^{2}+\frac{1}{\mathrm{LC}_{s}}}{\mathrm{Cp}_{\mathrm{p}}+\mathrm{C}_{\mathrm{s}}} \frac{L\left(\mathrm{CpC}_{\mathrm{s}}\right)}{}
$$

we see that the crystal has two resonant frequencies:

$$
\omega_{\mathrm{S}}=\frac{1}{\sqrt{\mathrm{LC}_{S}}} \text { and } \quad \omega \mathrm{p}=\frac{1}{\sqrt{\frac{\mathrm{C}_{S} \mathrm{Cp}}{\mathrm{C}_{S}+\mathrm{Cp}}}}
$$

they are series resonance and parallel resonance.

For $\mathrm{s}=\mathrm{j} \omega$

$$
Z(\mathrm{j} \omega)=-\mathrm{j} \frac{1}{\omega C p}\left(\frac{\omega^{2}-\omega^{2}}{\omega^{2}-\omega^{2}}\right)
$$

It can be seen that $\omega_{\mathrm{p}}>\omega_{\mathrm{s}}$, however, since $\mathrm{C}_{\mathrm{p}} \gg \mathrm{C}_{\mathrm{s}}$, the two resonant frequencies are very close.

From Figure 12.15, we observe that the crystal reactance is inductive over a narrow frequency band between $\omega_{\mathrm{p}}$ and $\omega_{\mathrm{s}}$. We may use the crystal to replace the inductor in a Colpitts oscillator. The resulting circuit will oscillate at the resonant frequency of the crystal inductance L with the series equivalent of $\mathrm{C}_{s}$ and $\left(\mathrm{Cp}+\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}\right)$.

Since $\mathrm{C}_{\mathrm{s}}$ is much smaller than the other capacitances, it will dominate and

$$
\omega=\frac{1}{\sqrt{\mathrm{LC}_{\mathrm{s}}}}=\omega
$$

A popular configuration of the Colpitts oscillator called Pierce oscillator is shown below.


Fig. 12.16 A Colpitts (or Pierce) crystal oscillator utilizing a CMOS inverter as an amplifier.

$$
C_{e g}=\frac{c_{s} \cdot\left[c_{p}+\frac{c_{1} c_{2}}{c_{1}+c_{2}}\right]}{c_{s}+c_{p}+\frac{c_{1} c_{2}}{c_{1}+c_{2}}}
$$

Resistor Rf determines the DC operating point in the high gain region of the CMOS inverter. Resistor R1 together with capacitor C1 provides a LPF that discourages the circuit from oscillating at higher harmonic of the crystal frequency.

Common to purchase crystal modules with TTL, CMOS or ECL outputs with 14 pin dip or surface mount. Crystals are available in standard frequencies and can be custom ordered for relatively low cost. The oscillators can be tuned a small amount with the use of variable capacitor (varactor) to create Voltage Control Crystal Oscillator (VCXO). Crystals are also be used in high Q filters such as crystal filters or SAW filters.

## ANALOG CIRCUITS UNIT-V ASSIGNMENT CUM TUTORIAL QUESTIONS <br> A. Questions testing his remembering / understanding level of students

 I. Objective/ Multiple Choice Questions1. When negative voltage feedback is applied to an amplifier, its voltage gain is [ ]
a) Decreases b) increases c) remains the same d) none of the above
2. The value of negative feedback fraction is always [ ]
a) Less than 1 b ) Greater than 1 c ) equal to 1 d ) none of the above
3. If the feedback fraction of an amplifier is 0.01 , then voltage gain with negative Feedback is approximately $\qquad$ [ ]
a) 10 b) 100 c) 1000 d) 0.01
4. The gain of an amplifier with feedback is known as $\qquad$ gain [ ]
b) Open loop gain b) closed loop gain c) Resonant d) none of the above
5. Negative feedback is employed in $\qquad$ [ ]
a) Oscillators b) Amplifiers c) Rectifiers d) None of the above
6. When a negative voltage feedback is applied to an amplifier, its bandwidth. $\qquad$ [ ]
a) Decreases b) increases c) remains the same d) none of the above
7. List some advantages of negative feedback amplifiers.
8. Gain increases with positive feedback [Y/N]
9. What would be the computational value of feedback voltage in a negative feedback Amplifier withA $=100, \beta=0.03$ and input signal voltage $=30 \mathrm{mV}$ [ ]
a) 0.03 b) 0.06 c) 0.09 d$) 0.15$
10. Define Desensitivity
II) Descriptive Questions
1.List the advantages of Negative Feedback and also draw the general structure of Feedback amplifiers with negative Feedback.
2.List the basic feedback topologies.
11. Whatis the type of feedback topology used in Voltage amplifier?. Draw its basic structure. 4. What is the type of feedback topology used in Current amplifier? Draw its basic structure. 5.What is the type of feedback topology used in Trans Conductance amplifier? Draw its basic structure.
12. What is the type of feedback topology used in Trans Resistance amplifier? Draw its basic structure.
7.What is Gain Margin \& Phase Margin? How they are for a stable Amplifier.
13. Draw the feedback structure and derive the expression for feedback gain with Negative Feedback.
14. Explain how the Negative feedback improve Gain Desensitization, and Bandwidth extension.
15. Derive expressions for input and output resistances of a series-shunt feedback amplifier.
16. Derive expressions for input and output resistances of a Shunt-Series feedback amplifier
17. Derive expressions for input and output resistances of a Series-Series feedback amplifier.
18. Derive expressions for input and output resistances of a series-shunt feedback amplifier.

Explain the effect of Feedback on Amplifier with Single- Pole response.

Derive expressions for input and output resistances of a Shunt-Shunt feedback amplifier B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. Emitter follower is a $\qquad$ circuit [ ]
a) Voltage feedback b) Current feedback c) Current \&Voltage feedback d) none of the above
2. If voltage feedback (negative) is applied to an amplifier, its output impedance $\qquad$ [ ]
a) Increases b) decreases c) remains same d) none of the above
3. For voltage amplifiers [ ]
a) Rincreases, Ro increases b) Rincreases, $R_{0}$ decreasesc)Ridecreases, $R_{0}$ increasesd) Ridecreases, $R_{o}$ decreases
4. For Current amplifiers [ ]
a) Rincreases, $R_{0}$ increases b) Rincreases, $R_{0}$ decreasesc) $R_{i}$ decreases, $R_{o}$ increases d) $\mathrm{R}_{\mathrm{i}}$ decreases, Rodecreases
5. For Trans conductance amplifiers [ ]
a) Rincreases, Ro increases
b) Rincreases, Ro decreasesc)Ridecreases, Ro increases d) $\mathrm{R}_{\mathrm{i}}$ decreases, $\mathrm{R}_{\mathrm{o}}$ decreases
6. For Trans resistance amplifiers [ ]
a) Rincreases, $R_{0}$ increases b) Rincreases, $R_{0}$ decreasesc) $R_{i} d$ decreases, $R_{o}$ increases d) Ridecreases, Rodecreases
7. Which type of amplifier is a voltage controlled voltage source [ ]
a) voltage amplifiers b) Current amplifiers c) Trans conductance d) Trans resistance
8. Which type of amplifier is a voltage controlled voltage source [ ]
a) voltage amplifiers b) Current amplifiers c) Trans conductance d) Trans resistance
9. Which type of amplifier is a voltage controlled voltage source [ ]
a) voltage amplifiers b) current amplifiers c) Trans conductance d) Trans resistance
10. Which type of amplifier is a voltage controlled voltage source [ ]
a) voltage amplifiers b) current amplifiers c) Trans conductance d) Trans resistance

## II. PROBLEMS

1. ) Explain how the application of Negative feedback reduces Interference and Non-linear Distortion. b) A feedback amplifier has a Forward Gain of $\mathrm{A}=1000 \& \beta=0.001$. Find the amount of feedback, Closed loop Gain Af .
2. A series-shunt feedback amplifier employs a basic amplifier with input and output resistance of $1 \mathrm{k} \Omega$ each and gain $A=2000$. If feedback factor $\beta=0.1$, find $A f$, Rif, Rof of closed loop amplifier.
3. A Shunt-Series feedback amplifier employs a basic amplifier with input and output resistance of $1 \mathrm{k} \Omega$ each and gain $A=1000$. If feedback factor $\beta=0.002$, find $A f$, Rif, Rof of closed loop amplifier.
4. A series-series feedback amplifier employs a basic amplifier with input and output resistance of $15 \mathrm{k} \Omega$ each and gain $A=2000$. If feedback factor $\beta=0.1$, find Af, Rif, Rof of closed loop amplifier.
5. A series-shunt feedback amplifier employs a basic amplifier with input and output resistance of $10 \mathrm{k} \Omega$ each and gain $A=2000$. If feedback factor $\beta=0.01$, find $A f$, Rif, Rof of closed loop amplifier.
6. An op amp having a single-pole roll off at 5000 Hz and a low-frequency gain of 200 is operated in a feedback loop with $\beta=0.01$. What is the factor by which Feedback Shifts pole? To What Frequency?
7. A feedback amplifier has a Forward Gain of $\mathrm{A}=2000 \& \beta=0.01$. Find a) Closed loop Gain Af, b) Amount of Feedback \& c) If A decreases by $10 \%$ what is the corresponding decrease in Af.

## D.Previous GATE/ IES Questions

1. The feedback amplifier shown in figure has [ ]\{2000\}

a) Current series feedback with large input impedance and large output impedance
b) Voltage series feedback with large input impedance and large output impedance
c) Voltage shunt feedback with large input impedance and large output impedance
d) Current shunt feedback with large input impedance and large output impedance
2. Negative feedback in amplifiers [ ] \{2004\}
a) Improves signal to noise ratio at the input
b) Improves signal to noise ratio at the output
c) Reduces distortion
d) Does not affect the signal to noise ratio at the output
3. To obtain high input, output impedances .......type of feedback topology is used [ ] \{2002\}
a) Voltage series b) voltage shunt c) current series d) current shunt
4. Negative feedback amplifier is an amplifier [ ] \{2009\}
a) Reduces gain b) reduces bandwidth c) increases noise d) increases frequency and phase distortions
5. In a shunt negative type of feedback amplifiers [ ] \{2011\}
a) High input impedance and high output impedance
b) high input impedance and low output impedance
c) low input impedance and low output impedance
d) low input impedance and high output impedance

## A. Questions testing his remembering / understanding level of students

## I. Objective/ Multiple Choice Questions

1. Only the condition $\mathrm{A} \beta=$ $\qquad$ must be satisfied for self-sustained [ ]

Oscillations to result.
A) 0
B) 1
C) -1
D) none of the above
2. Sinusoidal oscillators operate with $\qquad$ feedback.
3. Which of the following is required for oscillation?
A) $\mathrm{A} \beta>1$
B) The phase shift around the feedback network must be $180^{\circ}$.
C) Both A $\beta>1$ and the phase shift around the feedback network must be $180^{\circ}$.
D) None of the above
4. In order to start up, a feedback oscillator requires [ ]
A) Negative feedback less than 1
B) Positive feedback greater than 1
C) Unity feedback equal to 1
D) No feedback
5. Which of the following oscillators is (are) tuned oscillators?
A) Colpitts B) Hartley C) Crystal D) All of the above
6. One condition for positive feedback is that the phase shift around the feedback loop must be ------------.degrees.
7. An input signal is needed for an oscillator to start [T/F]
8. At series resonance, the impedance of a crystal is [ ]
A) Minimum B) Maximum C) zero D) one
9. Define an Oscillator.
10. At Parallel resonance, the impedance of a crystal is
A) Minimum B) Maximum C) zero D) one
11. Which of the following is an audio oscillator? [ ]
A) Wein bridge B) Hartley
C) Collpits
D) Crystal

## II) Descriptive Questions

1) Explain about the Barkhausen Criterion and also detail the dependency of oscillator frequency on slope of the phase response.
2) Explain about the Non linear Amplitude control.
3) Derive the expression for frequency of oscillations and the condition for starting oscillations of a wein-bridge oscillator.
4) Explain the operation of RC Phase shift oscillator and derive the expression for frequency of oscillations.
5) Explain the operation of Quadrature oscillator and derive the expression for frequency of oscillations.
6) Derive the expression for frequency of oscillations and the condition for starting oscillations of a Hartley oscillator.
7) Derive the expression for frequency of oscillations and the condition for starting oscillations of Collpits oscillator.
8) Explain in detail about Crystal Oscillator.

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. An oscillator converts .................
A. AC power into DC power
B. DC power into AC power
C. Mechanical power into DC power
D. None of the above
2. In Wein bridge oscillator frequency of oscillation is $\qquad$
A. Inversely proportional to RC
B. Directly proportional to RC
C. Doesn't depends on RC
D. None of the above
3. In an $L C$ oscillator, if the value of $L$ is increased four times, the frequency of oscillations is $\qquad$
A. Increased 2 times
B. Decreased 2 times
C. Increased 4 times
D. Decreased 4 times
4. In an wein bridge oscillator, if the value of $R$ is increased four times, the frequency
$\qquad$
A. Increases
B. Decreases
C. Remains the same
D. Insufficient data
5. If the crystal frequency increases with temperature, we say that crystal has $\qquad$ temperature coefficient.
A. Positive
B. negative
C. zero
D. none
6. At series or parallel resonance, the circuit behaves as a load
A. Capacitive
B. Resistive
C. Inductive
D. None of the above

## II) Problems:

1) For the popular limiter circuit with $\mathrm{V}=15 \mathrm{~V}, \mathrm{R}_{1}=30 \mathrm{k} \Omega, \mathrm{R}_{\mathrm{f}}=60 \mathrm{k} \Omega, \mathrm{R}_{2}=\mathrm{R}_{5}=9 \mathrm{k} \Omega$, and $\mathrm{R}_{3}=\mathrm{R}_{4}=3 \mathrm{k} \Omega$, find the limiting levels and the value of $\mathrm{V}_{\mathrm{I}}$ at which the limiting levels are reached. Also determine the limiter gain and the slope of the transfer characteristic in the positive and negative limiting regions. Assume that $\mathrm{V}_{\mathrm{D}}=0.7 \mathrm{~V}$.
2.) Consider a sinusoidal oscillator formed of an amplifier with a gain of 2 and a second-order band pass filter. Find the pole frequency and the center-frequency gain of the filter needed to produce sustained oscillations at 1 kHz
3.) For the circuit of Wien-bridge oscillator with a limiter used for amplitude control (a) Disregarding the limiter circuit, find the location of the closed-loop poles. (b) Find the frequency of oscillation. (c) With the limiter in place, find the amplitude of the output sine wave (assume that the diode drop is 0.7 V ).
4.) Using a BJT biased at $\mathrm{I}_{\mathrm{C}}=1 \mathrm{~mA}$, design a Colpitts oscillator to operate at $\omega_{0}=106 \mathrm{rad} / \mathrm{s}$. Use $\mathrm{C}_{1}=0.01 \mu \mathrm{~F}$ and assume that the coil available has a Q of 100 (this can be represented by a resistance in parallel with $\mathrm{C}_{1}$ given by $\mathrm{Q} / \omega_{0} \mathrm{C}_{1}$ ). Also assume that there is a load resistance at the collector of $2 \mathrm{k} \Omega$ and that for the BJT, ro $=100 \mathrm{k} \Omega$. Find $\mathrm{C}_{2}$ and L.
5.) A 2-MHz quartz crystal is specified to have $\mathrm{L}=0.52 \mathrm{H}, \mathrm{Cs}=0.012 \mathrm{pF}, \mathrm{Cp}=4 \mathrm{pF}$, and $\mathrm{r}=$ $120 \Omega$.. Find fs, fp, and Q.

## C. Question testing Analyzing/evaluating/Creative abilities of Student

1. An oscillator is formed by loading a trans conductance amplifier having a positive gain with a parallel RLC circuit and connecting the output directly to the input (thus applying positive feedback with a factor $\beta=1$ ). Let the trans conductance amplifier have an input resistance of $10 \mathrm{k} \Omega$ and an output resistance of $10 \mathrm{k} \Omega$.. The LC resonator has $\mathrm{L}=10 \mu \mathrm{H}, \mathrm{C}=1000 \mathrm{pF}$, and $\mathrm{Q}=100$. For what value of trans conductance Gm will the circuit oscillate? At what frequency?
2. For the circuit in Fig.1, find $\mathrm{L}(\mathrm{s}), \mathrm{L}(\mathrm{j} \omega)$, the frequency for zero loop phase, and for oscillation.


Figure 1
3. For the circuit in Fig.2, find $\mathrm{L}(\mathrm{s}), \mathrm{L}(\mathrm{j} \omega)$, the frequency for zero loop phase, and for oscillation.


Figure 2
4. For the circuit in Fig.3, break the loop at node X and find the loop gain (working backward for simplicity to find Vx in terms of $\mathrm{V}_{\mathrm{o}}$ ). For $\mathrm{R}=10 \mathrm{k} \Omega$, find C and $\mathrm{R}_{\mathrm{f}}$ to obtain sinusoidal oscillations at 10 kHz .


Figure 3
5. Consider the circuit of Fig. 4 without the limiter. Break the feedback loop at $X$ and find the loop gain. To do this, it is easier to start at the output and work backward, finding the various currents and voltages, and eventually $\mathrm{V}_{\mathrm{x}}$ in terms of $\mathrm{V}_{\mathrm{o}}$.


Figure 4
6. Use the expression derived in Q.no 5 to find the frequency of oscillation $f_{0}$ and the minimum required value of $\mathrm{R}_{\mathrm{f}}$ for oscillations to start in the circuit of Fig. 4.

## D. Previous GATE/ IES Questions

1. The below figure shows an RC phase shift oscillator (GATE 2011)


Solve the network and find the possible value of $\mathrm{h}_{\mathrm{fe}}$ for the transistor for oscillations to be possible. Also determine the frequency of such oscillations. Take $\mathrm{C}=0.01 \mu \mathrm{~F}$ and $\mathrm{h}_{\mathrm{ie}}=2 \mathrm{k} \Omega$.
2. Match the following (GATE 2012)
a) Hartley

1) Low frequency Oscillator
b) Collpits
2) High frequency Oscillator
c) Crystal
3) Stable frequency Oscillator
4) Relaxation frequency Oscillator
5) Negative resistance Oscillator
3. Calculate the frequency at which the zero transmission is obtained from the Wein Bridge shown below


In the circuit shown below, capacitors $\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ are very large and acts as short at input frequency, $\mathrm{V}_{\mathrm{i}}$ is a small signal input, The gain magnitude at $10 \mathrm{M} \mathrm{rad} / \mathrm{s}$ (GATE 2014)

# UNIT-VI <br> Output Stages and Power Amplifiers 

## Introducition:

An important function of the output stage is to provide the amplifier with a low output resistance so that it can deliver the output signal to the load without loss of gain. Since the output stage is the final stage of the amplifier, it usually deals with relatively large signals. Thus the smallsignal approximations and models either are not applicable or must be used with care.

The most challenging requirement in the design of an output stage is for it to deliver the required amount of power to the load in an efficient manner. This implies that the power dissipated in the output-stage transistors must be as low as possible.
This requirement comes mainly from the fact that the power dissipated in a transistor raises its internal junction temperature, and there is a maximum temperature (in the range of $150^{\circ} \mathrm{C}$ to $200^{\circ} \mathrm{C}$ for silicon devices) above which the transistor is destroyed.

A power amplifier is simply an amplifier with a high-power output stage.

## Classification of Output Stages:

Output stages are classified according to the collector current waveform that results when aninput signal is applied.

The transistor in a class A stage conducts for the entire cycle of the input signal; that is, the conduction $360^{\circ}$ as shown in $\mathrm{Fig}(\mathrm{a})$.

The transistor in class B stage is biased at zero dc current i.e in cut off. Thus a transistor in a class B stage conducts for only half the cycle of the input sine wave, resulting in a conduction angle of $180^{\circ}$ as shown in Fig(b).

(a)

(b)

An intermediate class between A and B, appropriately named class AB, involves biasing the transistor at a nonzero dc current much smaller than the peak current of the sine-wave signal. As a result, the transistor conducts for an interval slightly greater than half a cycle, as illustrated in Fig.(c). The resulting conduction angle is greater than $180^{\circ}$ but much less than $360^{\circ}$.

The transistor in Class $\mathbf{C}$ conducts for an interval shorter than that of a half-cycle; that is, the conduction angle is less than $180^{\circ}$ as in Fig (d). The result is the periodically pulsating current waveform shown. To obtain a sinusoidal output voltage, this current is a parallel LC circuit, tuned to the frequency of the input sinusoid.

(c)

(d)

Fig 1. Collector current waveforms for transistors operating in (a)class A, (b)class B, (c)class AB , and (d)class C amplifier stages.

Note: Class A, AB, and B amplifiers are studied in this chapter. They are employed as output stages of op amps and audio power amplifiers. In the latter application, class AB is the pre-ferred choice, for reasons that will be explained in the sections to follow. Class C amplifiers are usually employed for radio-frequency (RF) power amplification (required, e.g., in mobile phones and radio and TV transmitters)

## Class A Output Stage:

The most popular class a output stage uses emitter follower because of its low output resistance.
An emitter follower Q1 biased with a constant current I supplied by transistorQ2. Since the emitter current $i_{E I}=\mathrm{I}+\mathrm{i}_{\mathrm{L}}$, the bias current I must be greater than the largest negative load current; otherwise, Q1 cuts off and class A operation will no longer be maintained.


Fig. An emitter follower (Q1)biased with a constant current I supplied by transistor Q2
The transfer characteristic of the emitter follower is described by

$$
\begin{equation*}
v_{O}=v_{I}-v_{B E 1} \tag{1}
\end{equation*}
$$

the linear transfer curve is shown below, the positive limit of the linear region is determined by the saturation of Q1; thus

$$
\begin{equation*}
v_{O \max }=V_{C C}-V_{C E 1 \mathrm{sat}} \tag{2}
\end{equation*}
$$

In the negative direction, depending on the values of I and $\mathrm{R}_{\mathrm{L}}$, the limit of the linear region is determined either by Q1turning off or by Q2saturating,

$$
\begin{align*}
& v_{O \text { min }}=-I R_{L}  \tag{3}\\
& v_{O \text { min }}=-V_{C C}+V_{C E 2 \mathrm{sat}} \tag{4}
\end{align*}
$$



Fig. Transfer characteristic of the emitter follower

The absolutely lowest (most negative) output voltage is that given by Eq. (4) and is achieved provided the bias current $I$ is greater than the magnitude of the corresponding load current

$$
\begin{equation*}
I \geq \frac{\left|-V_{C C}+V_{C E 2 \text { sat }}\right|}{R_{L}} \tag{5}
\end{equation*}
$$

## Power-Conversion Efficiency:

The power-conversion efficiency of an output stage is defined as

$$
\begin{equation*}
\eta \equiv \frac{\text { Load power }\left(P_{L}\right)}{\text { Supply power }\left(P_{S}\right)} \tag{6}
\end{equation*}
$$

For the Claas- A using emitter follower assuming that the output voltage is a sinusoid with the peak value , $\hat{V}_{o}$, the average load power will be

$$
\begin{equation*}
P_{L}=\frac{\left(\hat{V}_{o} / \sqrt{2}\right)^{2}}{R_{L}}=\frac{1}{2} \frac{\hat{V}_{o}^{2}}{R_{L}} \tag{7}
\end{equation*}
$$

Since the current in Q 2 is constant ( I ), the power drawn from the negative supply is $\mathrm{V}_{\mathrm{CC}} * \mathrm{I}$. The average current in Q1 is equal to I, and thus the average power drawn from the positive supply is $\mathrm{V}_{\mathrm{CC}} * \mathrm{I}$. Thus the total average supply power is

$$
\begin{equation*}
P_{S}=2 V_{C C} I \tag{8}
\end{equation*}
$$

Equations (7) and (8) can be combined to yield

$$
\begin{aligned}
\eta & =\frac{1}{4} \frac{\hat{V}_{o}^{2}}{I R_{L} V_{C C}} \\
& =\frac{1}{4}\left(\frac{\hat{V}_{o}}{I R_{L}}\right)\left(\frac{\hat{V}_{o}}{V_{C C}}\right)
\end{aligned}
$$

Since $\hat{V}_{o} \leq V_{C C}$ and $\hat{V}_{o} \leq I R_{L}$, maximum efficiency is obtained when

$$
\hat{V}_{o}=V_{C C}=I R_{L}
$$

## Note:

The maximum efficiency attainable is $25 \%$. Because this is a rather low figure, the class A output stage is rarely used in high-power applications (>1 W).

## Class B Output Stage:

Class B Output Stage consists of a complementary pair of transistors (an npn and a pnp) connected in such a way that both cannot conduct simultaneously. Hence called as Complementary Symmetric Configuration.

Circuit Operation:
When the input voltage $\mathrm{v}_{\mathrm{I}}$ is zero, both transistors are cut off and the output voltage vo is zero. As $\mathrm{v}_{\mathrm{I}}$ goes positive and exceeds about $0.5 \mathrm{~V}, \mathrm{Q}_{\mathrm{N}}$ conducts and operates as an emitter follower. In this case vo follows $\mathrm{v}_{\mathrm{I}}$ (i.e., $\mathrm{v}_{\mathrm{O}}=\mathrm{v}_{\mathrm{I}}-\mathrm{v}_{\mathrm{BEN}}$ ) and $\mathrm{Q}_{\mathrm{N}}$ supplies the load current. Meanwhile, the emitterbase junction of $\mathrm{Q}_{\mathrm{P}}$ will be reverse-biased by the $\mathrm{V}_{\mathrm{BE}}$ of $\mathrm{Q}_{\mathrm{N}}$, which is approximately 0.7 V . Thus $\mathrm{Q}_{\mathrm{p}}$ will be cut off.


Fig. A class B output stage

If the input goes negative by more than about $0.5 \mathrm{~V}, Q_{P}$ turns on and acts as an emitter follower. Again $v_{o}$ follows $v_{I}$ (i.e., $v_{O}=v_{I}+v_{E B P}$ ), but in this case $Q_{P}$ supplies the load current and $Q_{N}$ will be cut off.


## Transfer Characteristic:

There exists a range of $v_{I}$ centered around zero where both transistors are cut off and $v_{0}$ is zero. This dead band results in the crossover distortion.

Fig.: Transfer characteristic for the class B output stage.

Fig: Illustrating how the dead band in the class B transfer characteristic results in crossover distortion.

## Power-Conversion Efficiency:



To calculate the power-conversion efficiency, $\boldsymbol{\eta}$, of the class $B$ stage, we neglect the crossover distortion and consider the case of an output sinusoid of peak amplitude $\hat{V}_{o}$.The average load power will be,

$$
\begin{equation*}
P_{L}=\frac{1 \hat{V}_{o}^{2}}{2} \frac{R_{L}}{R_{L}} \tag{1}
\end{equation*}
$$

The current drawn from each supply will consist of half-sine waves of peak amplitude $\left(\hat{V}_{o} / R_{L}\right)$. Thus the average current drawn from each of the two power supplies will be $\hat{V}_{o} / \pi R_{L}$. It follows that the average power drawn from each of the two power supplies will be the same,

$$
\begin{equation*}
P_{S+}=P_{S-}=\frac{1}{\pi} \frac{\hat{V}_{o}}{\pi R_{L}} V_{C C} \tag{2}
\end{equation*}
$$

and the total supply power will be

$$
\begin{equation*}
P_{S}=\frac{2 \hat{V}_{o}}{\pi R_{L}} V_{C C} \tag{3}
\end{equation*}
$$

Thus the efficiency will be given by $\eta=\left(\frac{1}{2} \frac{\hat{V}_{o}^{2}}{R_{L}}\right) /\left(\frac{2}{\pi} \frac{\hat{V}_{o}}{R_{L}} V_{C C}\right)=\frac{\pi}{4} \frac{\hat{V}_{o}}{V_{C C}}$

It follows that the maximum efficiency is obtained when $\hat{V}_{o}$ is at its maximum. This maximum is limited by the saturation of $Q_{N}$ and $Q_{P}$ to $V_{C C}-V_{C E s a t} \simeq V_{C C}$ At this value of peak output voltage, the power-conversion efficiency is

$$
\begin{equation*}
\eta_{\max }=\frac{\pi}{4}=78.5 \% \tag{5}
\end{equation*}
$$

This value is much larger than that obtained in the class A stage (25\%). the maximum average power available from a class B output stage is obtained by substituting $\hat{V}_{o}=V_{C C}$

$$
\begin{equation*}
P_{L \max }=\frac{1}{2} \frac{V_{C C}^{2}}{R_{L}} \tag{6}
\end{equation*}
$$

## Power Dissipation:

Unlike the class A stage, which dissipates maximum power under quiescent conditions ( $\mathrm{v}_{\mathrm{O}}=0$ ), the quiescent power dissipation of the class B stage is zero. When an input signal is applied, the average power dissipated in the class B stage is given by,

$$
\begin{equation*}
P_{D}=P_{S}-P_{L} \tag{7}
\end{equation*}
$$

Substituting for $\mathrm{P}_{\mathrm{S}}$ from Eq. (3) and for $\mathrm{P}_{\mathrm{L}}$ from Eq. (1) results in

$$
\begin{equation*}
P_{D}=\frac{2}{\pi} \frac{\hat{V}_{o}}{R_{L}} V_{C C}-\frac{1}{2} \frac{\hat{V}_{o}^{2}}{R_{L}} \tag{8}
\end{equation*}
$$

Differentiating Eq. (8) with respect to $\hat{V}_{o}^{\prime}$ and equating the derivative to zero gives the value of that results in maximum average power dissipation as

$$
\begin{equation*}
\left.\hat{V}_{o}\right|_{P_{D \max }}=\frac{2}{\pi} V_{C C} \tag{9}
\end{equation*}
$$

Substituting this value in Eq. (8) gives,

$$
\begin{equation*}
P_{D \max }=\frac{2 V_{C C}^{2}}{\pi^{2} R_{L}} \tag{10}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
P_{D N \max }=P_{D P \max }=\frac{V_{C C}^{2}}{\pi^{2} R_{L}} \tag{11}
\end{equation*}
$$

At the point of maximum power dissipation, the efficiency can be evaluated by substituting for $\hat{V}_{o}{ }^{\prime}$ from Eq. (9) into Eq. (5); hence, . $\eta=50 \%$.

The figure below shows a sketch of $\mathrm{P}_{\mathrm{D}}(8)$ versus the peak output voltage $\hat{V}_{o}$. It shows that increasing $\hat{V}_{o}$ beyond decreases the power dissipated in the class B stage while increasing the load power. The price paid is an increase in nonlinear distortion as a result of approaching the saturation region of operation of QN and QP. Transistor saturation flattens the peaks of the output sine waveform. Unfortunately, this type of distortion cannot be significantly reduced by the application of negative feedback, and thus transistor saturation should be avoided in applications requiring low THD.

## Reducing Crossover Distortion:

The crossover distortion of a class B output stage can be reduced substantially by employing a high-gain op amp and overall negative feedback, as shown in Fig. below. The $\pm 0.7-\mathrm{V}$ dead band is reduced to volt, where A0is the dc gain of the op amp. Nevertheless, the slew-rate limitation of the op amp will cause the alternate turning on and off of the output transistors to be noticeable, especially at high frequencies. A more practical method for reducing and almost eliminating crossover distortion is found in the class AB stage, which will be studied in the next section.


Fig. Class B circuit with an op amp connected in a negative-feedback loop to reduce crossover distortion.

## Single-Supply Operation

The class B stage can be operated from a single power supply, in which case the load is capacitive coupled, to make the formulas derived in Dual Supply case directly applicable, the single power supply
 is denoted $2 \mathrm{~V}_{\mathrm{CC}}$

Fig. Class B output stage operated with a single power supply.

## Class AB Output Stage:

Crossover distortion can be virtually eliminated by biasing the complementary output transistors at a small nonzero current. The result is the class AB output stage shown in Figure below, A bias voltage $\mathrm{V}_{\mathrm{BB}}$ is applied between the bases of QN and QP . For vI $=0$, vO $=0$, and a voltage $V_{B B} / 2$ appears across the base-emitter junction of each of QN and QP
. Assuming matched devices

$$
\begin{equation*}
i_{N}=i_{P}=I_{Q}=I_{S} e^{V_{B B} / 2 V_{T}} \tag{1}
\end{equation*}
$$

The value of $\mathrm{V}_{\mathrm{BB}}$ is selected to yield the required quiescent current $\mathrm{I}_{\mathrm{Q}}$

## Circuit Operation:

When vI goes positive by a certain amount, the voltage at the base of by thesame amount and the output becomes positive at an almost equal value,

$$
\begin{equation*}
v_{O}=v_{I}+\frac{V_{B B}}{2}-v_{B E N} \tag{2}
\end{equation*}
$$



$$
v_{B E N}+v_{E B P}=V_{B B}
$$

## Fig. Class AB output stage

The positive vO causes a current $\mathrm{i}_{\mathrm{L}}$ to flow through $\mathrm{R}_{\mathrm{L}}$
, and thus $i_{N}$ must increase; that is,

$$
\begin{equation*}
i_{N}=i_{P}+i_{L} \tag{3}
\end{equation*}
$$

The increase in $i_{N}$ will be companied by a corresponding increase in $\mathrm{v}_{\mathrm{ben}}$ (above the quiescent value of $\mathrm{VBB} / 2$ ).

$$
V_{T} \ln \frac{i_{N}}{I_{S}}+V_{T} \ln \frac{i_{P}}{I_{S}}=2 V_{T} \ln \frac{I_{Q}}{I_{S}}
$$

$$
i_{N} i_{P}=I_{Q}^{2}
$$

Thus, as iN increases, iP decreases by the same ratio while the product remains constant. Equations (3) and (4) can be combined to yield iN for a given iL as the solution to the quadratic equation

$$
\begin{equation*}
i_{N}^{2}-i_{L} i_{N}-I_{Q}^{2}=0 \tag{5}
\end{equation*}
$$

From the equations above, we can see that for positive output voltages, the load current is supplied by QN, which acts as the output emitter follower. Meanwhile, QP will be conducting a current that decreases as vO increases; for large current in QP can be ignored altogether.
For negative input voltages the opposite occurs: The load current will be supplied by QP,which acts as the output emitter follower, while QN conducts a current that gets smaller as more negative.
the class AB stage operates in much the same manner as the class B circuit, with one important exception: For small vI, both transistors conduct, and as vI is increased or decreased, one of the two transistors takes over the operation. Since the transition is a smooth one, crossover distortion will be almost totally
 eliminated.

Fig. Transfer characteristic of the class AB stage

## Output Resistance:

If we assume that the source supplying vI is ideal, then the output resistance of the class AB stage can be determined

$$
\begin{equation*}
R_{\mathrm{out}}=r_{e N} \| r_{e P} \tag{1}
\end{equation*}
$$

wherere N and Pare the small-signal emitter resistances of QNand QP, respectively. At a given input voltage, the currents iN and iP can be determined, and reNand rePare given by


$$
\begin{aligned}
& r_{e N}=\frac{V_{T}}{i_{N}} \\
& r_{e P}=\frac{V_{T}}{i_{P}}
\end{aligned}
$$

Thus,

$$
R_{\mathrm{out}}=\frac{V_{T}}{i_{N}} \| \frac{V_{T}}{i_{P}}=\frac{V_{T}}{i_{P}+i_{N}}
$$

## Biasing the Class AB Circuit:

There are two approaches for generating the voltage VBB required for biasing the class AB output stage.

## 1. Biasing Using Diodes

2. Biasing Using the VBE Multiplier

## 1. Biasing Using Diodes:

A class AB circuit in which the bias voltage VBB is generated by passing a constant current I BIAS through a pair of diodes, or diode-connected transistors, D1 and D2. The biasing diodes, however, need not be large devices, and thus the quiescent current IQ established in QN and QP will be $\mathrm{IQ}=\mathrm{n}^{*}$ IBIAS, where n is the ratio of the emitter-junction area of the output devices to the junction area of the biasing diodes.


## Fig. : A class AB output stage utilizing diodes for biasing

When the output stage of Fig. 11.14 is sourcing current to the load, the base current of QN increases from (which is usually small)to approximately. This base current drive must be supplied by the current source Ibias. It follows that $\mathrm{I}_{\text {bias }}$ must be greater than the maximum anticipated base drive for QN. This sets a lower limit on the value of $\mathrm{I}_{\text {BIAs. }}$. Now, since $\mathrm{n}=\mathrm{IQ} / \mathrm{I}_{\mathrm{BIAS}}$, and since IQ is usually much smaller than the peak load current $(<10 \%)$, it is clear than cannot be a large number. This is a disadvantage of the diode biasing scheme
The diode biasing arrangement has an important advantage: It can provide thermal stabilization of the quiescent current in the output stage. The class AB output stage dissipates power under quiescent conditions. Power dissipation raises the internal temperature of the BJTs. If VBE is held constant and the temperature increases, the collector current increases. The increase in collector current increases the power dissipation, which in turn increases the junction temperature and hence, once more, the collector current. Thus a positive-feedback mechanism exists that can result in a phenomenon called thermal runaway.

Diode biasing can be arranged to provide a compensating effect that can protect the output transistors against thermal runaway under quies-cent conditions. Specifically, if the diodes are in close thermal contact with the output transistors, their temperature will increase by the same amount as that of QNand QP. Thus VBB will decrease, with the result that IQ remains constant. Close thermal contact is easily achieved in IC fabrication. It is obtained in discrete circuits by mounting the bias diodes on the metal case of QNor QP.

## 2. Biasing Using the VBEMultiplier:

An alternative biasing arrangement that provides the designer with considerably more flexi-bility in both discrete and integrated designs is shown in Fig. 11.15. The bias circuit consists of transistor Q1with a resistor R1connected between base and emitter and a feedback resistor R2connected between collector and base. The resulting two-terminal network is fed with a constant-current source $\mathrm{I}_{\text {bias. }}$ If we neglect the base current of Q1, then R1and R2will carry the same current $\mathrm{I}_{\mathrm{R}}$, given by

## Fig. A class AB output stage utilizing a VBE multiplier for biasing



Thus the circuit simply multiplies VBE1by the factor and is known as the "VBEmultiplier." The

$$
I_{R}=\frac{V_{B E 1}}{R_{1}}
$$

and the voltage $V_{B B}$ across the bias network will be

$$
\begin{aligned}
V_{B B} & =I_{R}\left(R_{1}+R_{2}\right) \\
& =V_{B E 1}\left(1+\frac{R_{2}}{R_{1}}\right)
\end{aligned}
$$

## Power BJTs:

Transistors that are required to conduct currents in the ampere range and to withstand power dissipation in the watts and tens-of-watts ranges differ in their physical structure, packaging, and specification from the small-signal transistors.

The important properties of power transistors are,

## 1. Junction Temperature 2. Thermal Resistance 3. Power Dissipation Versus Temperature (or Power Derating curve)

## 1. Junction Temperature:

Power transistors dissipate large amounts of power in their collector-base junctions. The dissipated power is converted into heat, which raises the junction temperature. However, the
junction temperature TJmust not be allowed to exceed a specified maximum, TJmax; other-wise the transistor could suffer permanent damage. For silicon devices, TJmaxis in the range of $150^{\circ} \mathrm{C}$ to $200^{\circ} \mathrm{C}$.

## 2. Thermal Resistance:

Consider first the situation of a transistor operating in free air-that is, with no special arrangements for cooling. The heat dissipated in the transistor junction will be conducted away from the junction to the transistor case, and from the case to the surrounding environ-ment. In a steady state in which the transistor is dissipating PD watts, the temperature rise of the junction relative to the surrounding ambience can be expressed as

$$
\begin{equation*}
T_{J}-T_{A}=\theta_{J A} P_{D} \tag{1}
\end{equation*}
$$

where $\theta$ JAis the thermal resistance between junction and ambience, having the units of degrees Celsius per watt. Note that $\theta$ JA simply gives the rise in junction temperature over the ambient temperature for each watt of dissipated power. Since we wish to be able to dissipatelarge amounts of power without raising the junction temperature above TJmax, it is desirable to have, for the thermal resistance $\theta \mathrm{JA}$, as small a value as possible. For operation in free air, $\theta \mathrm{JA}$ depends primarily on the type of case in which the transistor is packaged. The value of $\theta \mathrm{JA}$ is usually specified on the transistor data sheet.

Equation (1), which describes the thermal-conduction process, is analogous to Ohm's law, which describes the electrical-conduction process. In this analogy, power dissipation corresponds to current, temperature difference corresponds to voltage difference, and thermal resistance corresponds to electrical resistance. Thus, we may represent the thermal-conduction process by the electric circuit shown in Fig. below.


Fig. Electrical equivalent circuit of the thermal-conduction process; $\mathrm{TJ}-\mathrm{TA}=\mathrm{P}_{\mathrm{D}} \theta \mathrm{JA}$.

## 3. Power Dissipation Versus Temperature:

The transistor manufacturer usually specifies the maximum junction temperature TJmax , the maximum power dissipation at a particular ambient temperature TA0(usually, $25^{\circ} \mathrm{C}$ ), and the


Fig: Maximum allowable power dissipation versus ambient temperature for a BJT operated infree air. This is known as a "power-derating" curve
thermal resistance $\theta \mathrm{JA}$. In addition, a graph such as that shown in Fig. above is usually provided. The graph simply states that for operation at ambient temperatures below TA0, thedevice can safely dissipate the rated value of PD0watts. However, if the device is to be oper-ated at higher ambient temperatures, the maximum allowable power dissipation must bederatedaccording to the straight line shown in Fig.above. The power-deratingcurveis agraphical representation of Eq. (1). Specifically, note that if the ambient temperature isTA0and the power dissipation is at the maximum allowed (PD0), then the junction temperaturewill be TJmax. Substituting these quantities in Eq. (1) results in

$$
\theta_{J A}=\frac{T_{J \max }-T_{A 0}}{P_{D 0}}
$$

which is the inverse of the slope of the power-derating straight line. At an ambient temperatureTA, higher than TA0, the maximum allowable power dissipation PDmaxcan be obtained from Eq. (1) by substituting TJ= TJmax; thus,

## Transistor Case and Heat Sink

The thermal resistance between junction and ambience, $\theta$ JA , can be expressed as

$$
\begin{equation*}
\theta_{J A}=\theta_{J C}+\theta_{C A} \tag{1}
\end{equation*}
$$

where $\theta \mathrm{JCis}$ the thermal resistance between junction and transistor case (package) and $\theta \mathrm{CA}$ isthe thermal resistance between case and ambience. For a given transistor, $\theta$ JCis fixed by the device design and packaging.

Although the circuit designer has no control over $\operatorname{VJC}$ (once a particular transistor hasbeen selected), the designer can considerably reduce $\theta$ CAbelow its free-air value (specifiedby the manufacturer as part of $\theta \mathrm{JA}$ ). Reduction of $\theta$ CAcan be effected by providing means tofacilitate
heat transfer from case to ambience. A popular approach is to bolt the transistor tothe chassis or to an extended metal surface. Such a metal surface then functions as a heat sink. Heat is easily conducted from the transistor case to the heat sink; that is, the thermal resistance $\theta$ CSis usually very small. Also, heat is efficiently transferred (by convection and radiation) from the heat sink to the ambience, resulting in a low thermal resistance $\theta$ SA. Thus, if a heat sink is utilized, the case-to-ambience thermal resistance given by

$$
\theta_{C A}=\theta_{C S}+\theta_{S A}
$$



Fig. Electrical analog of the thermal conduction process when a heat sink is utilized.


Fig. Maximum allowable power dissipation versus transistor-case temperature
The electrical analog of the thermal-conduction process when a heat sink is employed is

$$
T_{J}-T_{A}=P_{D}\left(\theta_{J C}+\theta_{C S}+\theta_{S A}\right)
$$

## ASSIGNMENT CUM TUTORIAL QUESTIONS

## A. Questions testing his remembering / understanding level of students

## I. Objective/ Multiple Choice Questions

1. The Q-point is at cutoff for class $\qquad$ power amplifier.
A) A
B) B
C) AB
D) C
2. What is the maximum efficiency of a class A circuit with a direct or series-fed load connection
A) $90 \%$
B) $78.5 \%$
C) $50 \%$
D) $25 \%$
3. Class AB operation is $\qquad$ operation. [ ]
A) Similar to Class A B) Similar to Class B
C) Similar to Class C . D) None of the above
4. In $\qquad$ power amplifiers, the output signal varies for a full $360^{\circ}$ of the cycle [ ]
A) Class A
B) Class B
C) Class C
. D) Class AB
5. In class B power amplifiers, the output signal varies for $\qquad$ of the cycle.
6. $\qquad$ power amplifier have the highest overall efficiency.
7. Cross over distortion occurs in $\qquad$ Power amplifier.
8. Define conversion Efficiency
9. Power amplifiers are large signal amplifiers. [True/False]
10. What is meant by Cross over distortion
11. Which of the following is/are biasing technique for Class AB power Amplifier [ ]
A) Biasing using diodes
B) Biasing using $\mathrm{V}_{\mathrm{BE}}$ Multiplier
C) Both A \& B
D) None
II) Descriptive Questions
12. a) Define Power Conversion Efficiency?
b) Draw Collector Current waveforms of Class A \& Class B , Class AB \& Class C Output Stages.
c) Explain Crossover distortion?
d) What is Thermal Resistance \& Draw its electrical equivalent circuit?
e) Explain the Thermal resistance property of Power BJTs.
13. a) A BJT is specified to have a maximum power dissipation $\mathrm{PD}_{0}$ of 5 W at an ambient temperature TA0 of $25^{\circ} \mathrm{C}$, and a maximum junction temperature $\mathrm{T}_{\mathrm{Jmax}}$ of $125^{\circ} \mathrm{C}$. Find thermal resistance $\theta_{\mathrm{JA}}$.
b) Draw the Electrical equivalent of Thermal conduction process when Heat Sink is used.
c) A class B output stage operates with Vcc=20V , $\mathrm{P}_{\mathrm{L}}=15 \mathrm{~W}$ \& average Supply power Ps $=25 \mathrm{~W}$. Find $\eta \%$, Maximum power dissipated at each transistor.
d) Explain the circuit for reducing cross-over distortion in Class- B Output stage.
14. Explain the Classification of Output Power amplifiers.
15. Detail the operation of Class A Output stage with derivation of Power Conversion efficiency.
16. Derive the expressions for Power Conversion Efficiency \& Power Dissipation of Class B Output Stage.
17. Explain the circuit for reducing cross-over distortion in Class- B Output stage.
18. a) Explain Class- AB output stage biasing using VBE Multiplier circuit.
b) Explain Class - AB output stage biasing using Diodes.
19. Explain the Significance of Power Derating curve in power BJTs specifications.
20. What is the significance of Heat Sink in power transistors? Explain with the electrical equivalent of Thermal conduction when Heat sink is used.
21. Draw the electrical equivalent of Thermal conduction when Heat sink is used. Explain each term in that.

## B. Question testing the ability of students in applying the concepts.

## I) Multiple Choice Questions:

1. An oscillator converts .................
A. AC power into DC power
B. DC power into AC power
C. Mechanical power into DC power
D. None of the above
2. In Wein bridge oscillator frequency of oscillation is $\qquad$
A. Inversely proportional to RC
B. Directly proportional to RC
C. Doesn't depends on RC
D. None of the above
3. In an LC oscillator, if the value of $L$ is increased four times, the frequency
$\qquad$
A. Increased 2 times
B. Decreased 2 times
C. Increased 4 times
D. Decreased 4 times

## 4. In an wein bridge oscillator, if the value of $\mathbf{R}$ is increased four times, the frequency

 of oscillations is $\qquad$B. Increases
B. Decreases
C. Remains the same
D. Insufficient data
5. If the crystal frequency increases with temperature, we say that crystal has $\qquad$ Temperature coefficient.
A. Positive
B. negative
C. zero
D. none
6. At series or parallel resonance, the circuit behaves as a $\qquad$ load
A. Capacitive
B. Resistive
C. Inductive
D. None of the above

## II) Problems:

1. For the Class-A emitter follower, $\mathrm{V}_{\mathrm{CC}}=20 \mathrm{~V}, \mathrm{~V}_{\mathrm{CEsat}}=0.2 \mathrm{~V}, \mathrm{~V}_{\mathrm{BE}}=0.7 \mathrm{~V}$ and constant, and $\beta$ is very high. Find i) the value of $R$ that will establish a bias current sufficiently large to allow the largest possible output signal swing for $\mathrm{R}_{\mathrm{L}}=5 \mathrm{k} \Omega$. ii) Determine the resulting output signal swing
2. For the Class- A Emitter follower, let $\mathrm{V}_{\mathrm{CC}}=12 \mathrm{~V}, \mathrm{I}=120 \mathrm{~mA}$, and $\mathrm{RL}=100 \Omega$. If the output voltage is an 8 -V-peak sinusoid, find the following: (a) the power delivered to the load; (b) the Average power drawn from the supplies; (c) the power-conversion efficiency. Ignore the loss inQ3and R.
3. Design a class B output stage to deliver an average power of 15 W to a $8-\Omega$ load. The power supply is to be selected such that $\mathrm{V}_{\mathrm{CC}}$ is about 5 V greater than the peak output voltage. Determine i) the supply voltage required ii) the peak current drawn from each supply, iii) the total supply power iv) the power-conversion efficiency. v) Determine the maximum power that each transistor must be able to dissipate safely
4. For the class B output stage, let $\mathrm{V}_{\mathrm{CC}}=12 \mathrm{~V}$ and $\mathrm{R}_{\mathrm{L}}=4 \Omega$. If the output is a sinusoid with8V peak amplitude, find (a) the output power; (b) the average power drawn from each supply; (c)the power efficiency obtained at this output voltage; (d) the peak currents supplied by $v_{I}$, assuming that $\beta_{\mathrm{N}}=\beta_{\mathrm{P}}=40$; (e) the maximum power that each transistor must be capable of dissipating safely
5. A BJT is specified to have a maximum power dissipation PD0of 4 W at an ambient temperature $\mathrm{T}_{\mathrm{A} 0}$ of $28^{\circ} \mathrm{C}$, and a maximum junction temperature $\mathrm{T}_{\mathrm{Jmax}}$ of $145^{\circ} \mathrm{C}$. Find the following:(a) The thermal resistance $\theta_{\mathrm{JA}}$. (b) The maximum power that can be safely dissipated at an ambient temperature of $35^{\circ} \mathrm{C}$.(c) The junction temperature if the device is operating at $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ and is dissipating 4 W .
6. A BJT is specified to have $\mathrm{T}_{\mathrm{Jmax}}=150^{\circ} \mathrm{C}$ and to be capable of dissipating maximum power as follows: 45 W at $\mathrm{Tc}=25^{\circ} \mathrm{C} ; 2 \mathrm{~W}$ at $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ Above $25^{\circ} \mathrm{C}$, the maximum power dissipation is to be derated linearly with $\theta_{\mathrm{JC}}=4.5^{\circ} \mathrm{C} / \mathrm{W}$ and $\theta_{\mathrm{JA}}=65.5^{\circ} \mathrm{C} / \mathrm{W}$. Find the following:(a) The maximum power that can be dissipated safely by this transistor when operated in free air at $\mathrm{T}_{\mathrm{A}}=55^{\circ} \mathrm{C}$.
7. (b) The maximum power that can be dissipated safely by this transistor when operated at an ambient temperature of $50^{\circ} \mathrm{C}$, but with a heat sink for which $\theta_{\mathrm{CS}}=0.4^{\circ} \mathrm{C} / \mathrm{W}$ and $\theta_{\mathrm{SA}}=5^{\circ} \mathrm{C} / \mathrm{W}$. Find the temperature of the case and of the heat sink.(c) The maximum power that can be dissipated safely if an infinite heat sink is used and $\mathrm{T}_{\mathrm{A}}=50^{\circ} \mathrm{C}$.

## C. Question testing his Analyzing/evaluating/Creative abilities of Student

1.) A class $B$ output stage is required to deliver an average power of 100 W into a 16 ohm load. The power supply should be 4 V greater than the corresponding peak sine-wave output voltage. Determine the power -supply voltage required, to the nearest volt in the appropriate direction, the peak current from each supply, the total supply power, and the power conversion efficiency. Also, determine the maximum possible power dissipation in each transistor for a sine-wave input.
2.) The design of a class AB MOS output stage is being considered. The available devices have $\left|\mathrm{V}_{\mathrm{n}}\right|=1 \mathrm{~V}$ and $\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \mathrm{W} / \mathrm{L}=200 \mathrm{~mA} / \mathrm{V}^{2}$. What value of gate-to-gate bias voltage, $\mathrm{V}_{\mathrm{GG}}$ is required to reduce the incremental output resistance in the quiescent state tol0 $\Omega$.
3.) A particular transistor having a thermal resistance $\theta_{\mathrm{JA}}=2^{0} \mathrm{C} / \mathrm{W}$ is operating at an ambient temperature of $30^{\circ} \mathrm{C}$ with a collector-emitter voltage of 20 V . If long life requires a maximum junction temperature of $130^{\circ} \mathrm{C}$, what is the corresponding device power rating? What is the greatest average collector current that should be considered?.
4.) A power transistor operating at an ambient temperature of $50^{\circ} \mathrm{C}$ and an average emitter current of 3 A , dissipates 30 W . If the thermal resistance of the transistor is known to be less than $3^{\circ} \mathrm{C} / \mathrm{W}$, what is the greatest junction temperature you would expect? If the transistor $\mathrm{V}_{\mathrm{BE}}$ measured using a pulsed emitter current of 3 A at a junction temperature of $25^{\circ} \mathrm{C}$ is 0.80 V , what average $\mathrm{V}_{\mathrm{BE}}$ would you expect under normal operating conditions? Use a temperature coefficient of $-2 \mathrm{mV} /{ }^{0} \mathrm{C}$.

## D. Previous GATE/ IES Questions

1. Cross over distortion behavior is a characteristic of (GATE 2011)
A) Class A
B) Class B
C) Class C . D) Class AB
2. A class A transformer coupled transistor power amplifier is required to deliver a power output of 10 watts. The maximum power rating of the transistor should not be less than (GATE 2012)
A) 5 watts B) 10 watts C) 15 watts D) 20 watts
3. A power amplifier delivers 50 watts output at $50 \%$ efficiency. The ambient temperature is $25^{\circ} \mathrm{C}$. If the maximum allowable junction temperature is $150^{\circ} \mathrm{C}$, the maximum thermal resistance $\theta_{\mathrm{JC}}$ that can be tolerated is
4. For the 2 N 338 , the manufacturer specifies $\mathrm{P}_{\max }=100 \mathrm{~mW}$ at $25^{\circ} \mathrm{C}$, free air temperature and the maximum junction temperature $\mathrm{T}_{\mathrm{jmax}}=125^{\circ} \mathrm{C}$. Its thermal resistance is -----------.(GATE 2004)
